

4721

Mark Scheme

January 2008

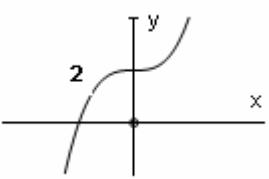
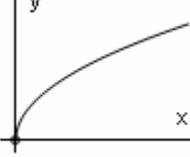
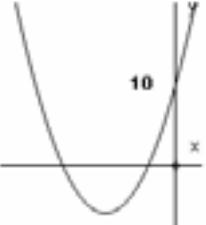
## 4721 Core Mathematics 1

|       |   |                      |   |
|-------|---|----------------------|---|
| 1     | $\begin{aligned} & \frac{4(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} \\ &= \frac{12+4\sqrt{7}}{9-7} \\ &= 6+2\sqrt{7} \end{aligned}$ | M1<br>B1<br>A1       | Multiply top and bottom by conjugate<br>$9 \pm 7$ soi in denominator<br>$6 + 2\sqrt{7}$<br><span style="border: 1px solid black; padding: 2px;">3</span>  |
| 2(i)  | $x^2 + y^2 = 49$  | B1                   | $x^2 + y^2 = 49$  |
| (ii)  | $x^2 + y^2 - 6x - 10y - 30 = 0$<br>$(x-3)^2 - 9 + (y-5)^2 - 25 - 30 = 0$<br>$(x-3)^2 + (y-5)^2 = 64$<br>$r^2 = 64$<br>$r = 8$     | M1<br>A1             | $3^2 \ 5^2 \ 30$ with consistent signs soi<br><span style="border: 1px solid black; padding: 2px;">2</span><br><span style="border: 1px solid black; padding: 2px;">3</span> cao                                      |
| 3     | $a(x+3)^2 + c = 3x^2 + bx + 10$<br>$3(x^2 + 6x + 9) + c = 3x^2 + bx + 10$<br>$3x^2 + 18x + 27 + c = 3x^2 + bx + 10$<br>$c = -17$  | B1<br>B1<br>M1<br>A1 | $a = 3$ soi<br>$b = 18$ soi<br>$c = 10 - 9a$ or $c = 10 - \frac{b^2}{12}$<br><span style="border: 1px solid black; padding: 2px;">4</span><br><span style="border: 1px solid black; padding: 2px;">4</span> $c = -17$ |
| 4(i)  | $p = -1$  | B1                   | $p = -1$  |
| (ii)  | $\sqrt{25k^2} = 15$<br>$25k^2 = 225$<br>$k^2 = 9$<br>$k = \pm 3$  | M1<br>A1<br>A1       | Attempt to square 15 or attempt to square root $25k^2$<br>$k = 3$<br>$k = -3$   |
| (iii) | $\sqrt[3]{t} = 2$<br>$t = 8$  | M1<br>A1             | $\frac{1}{t^{\frac{1}{3}}} = \frac{1}{2}$ or $t^{\frac{1}{3}} = 2$ soi<br><span style="border: 1px solid black; padding: 2px;">2</span><br><span style="border: 1px solid black; padding: 2px;">6</span> $t = 8$      |

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| 5(i)  |    | B1<br>B1 2                   | +ve cubic<br>+ve or -ve cubic with point of inflection at (0, 2) and no max/min points                                       |
| (ii)  |    | B1<br>B1 2                   | curve with correct curvature in +ve quadrant only<br>completely correct curve  |
| (iii) | Stretch<br>scale factor 1.5<br>parallel to y-axis   | B1<br>B1<br>B1 3<br><b>7</b> | stretch<br>factor 1.5<br>parallel to y-axis or in y-direction  |
| 6(i)  | EITHER<br>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-8 \pm \sqrt{64 - 40}}{2}$ $x = \frac{-8 \pm \sqrt{24}}{2}$ $x = \frac{-8 \pm 2\sqrt{6}}{2}$ $x = -4 \pm \sqrt{6}$<br><br>OR<br>$(x + 4)^2 - 16 + 10 = 0$ $(x + 4)^2 = 6$ $x + 4 = \pm\sqrt{6}$ $x = \pm\sqrt{6} - 4$ | M1<br><br>A1<br><br>A1 3     | Correct method to solve quadratic<br><br>$x = \frac{-8 \pm \sqrt{24}}{2}$<br><br>$x = -4 \pm \sqrt{6}$                       |
| (ii)  |    | B1<br>B1<br>B1 3             | +ve parabola<br>parabola cutting y-axis at (0, 10) where (0, 10) is not min/max point<br>parabola with 2 negative roots      |
| (iii) | $x \leq -\sqrt{6} - 4, x \geq \sqrt{6} - 4$   | M1<br>A1 ft 2<br><b>8</b>    | $x \leq \text{lower root } x \geq \text{higher root}$ (allow $<$ , $>$ )<br>Fully correct answer, ft from roots found in (i) |

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|---|----------------------------------|-------|---------------------------|---|
| 7(i)                                      | $\text{Gradient} = -\frac{1}{2}$ | B1 1  | $-\frac{1}{2}$            |   |
|   | $y - 5 = -\frac{1}{2}(x - 6)$    | M1    |                           | Equation of straight line through (6, 5) with any non-zero numerical gradient |
|   | $2y - 10 = -x + 6$               | B1 ft |                           | Uses gradient found in (i) in their equation of line                          |
|   | $x + 2y - 16 = 0$                | A1 3  |                           | Correct answer in correct form (integer coefficients)                         |
|   | EITHER                           | *M1   |                           | Substitute to find an equation in $x$ (or $y$ )                               |
|   | $\frac{4-x}{2} = x^2 + x + 1$    |       |                           |   |
|   | $4 - x = 2x^2 + 2x + 2$          | DM1   |                           | Correct method to solve quadratic   |
|   | $2x^2 + 3x - 2 = 0$              | A1    | $x = \frac{1}{2}, x = -2$ |   |
|   | $(2x-1)(x+2) = 0$                |       | $y = \frac{7}{4}, y = 3$  |   |
|   | $x = \frac{1}{2}, x = -2$        | A1 4  |                           | <b>SR</b> one correct (x,y) pair <b>www B1</b>                                |
| <b>OR</b>                                 |                                  |       |                           |   |
| $y = (4 - 2y)^2 + (4 - 2y) + 1 * M$       |                                  |       |                           |   |
| $y = 16 - 16y + 4y^2 + 4 - 2y + 1$        |                                  |       |                           |   |
| $0 = 21 - 19y + 4y^2$                     |                                  |       |                           |   |
| $0 = (4y - 7)(y - 3) \quad \text{DM1}$    |                                  |       |                           |   |
| $y = \frac{7}{4}, y = 3 \quad \text{A1}$  |                                  |       |                           |   |
| $x = \frac{1}{2}, x = -2 \quad \text{A1}$ |                                  |       |                           |   |

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| 8(i)  | $\frac{dy}{dx} = 3x^2 + 2x - 1$<br>At stationary points,<br>$3x^2 + 2x - 1 = 0$<br>$(3x - 1)(x + 1) = 0$<br>$x = \frac{1}{3}, x = -1$<br>$y = \frac{76}{27}, y = 4$ | *M1<br>A1<br>M1<br>DM1<br>A1<br>A1 6 | Attempt to differentiate (at least one correct term)<br>3 correct terms<br>Use of $\frac{dy}{dx} = 0$<br>Correct method to solve 3 term quadratic<br>$x = \frac{1}{3}, x = -1$<br>$y = \frac{76}{27}, 4$<br><b>SR</b> one correct (x,y) pair <b>www B1</b> |
| (ii)  | $\frac{d^2y}{dx^2} = 6x + 2$<br>$x = \frac{1}{3}, \frac{d^2y}{dx^2} > 0$<br>$x = -1, \frac{d^2y}{dx^2} < 0$   | M1<br>A1<br>A1 3                     | Looks at sign of $\frac{d^2y}{dx^2}$ for at least one of their x-values or other correct method<br>$x = \frac{1}{3}$ , minimum point <b>CWO</b><br>$x = -1$ , maximum point <b>CWO</b>   |
| (iii) | $-1 < x < \frac{1}{3}$  | M1<br>A1 2                           | Any inequality (or inequalities) involving both their x values from part (i)<br>Correct inequality (allow < or $\leq$ )<br><b>[1]</b>  |

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| 9(i)  | $\text{Gradient of AB} = \frac{-2-1}{-5-3}$<br>$= \frac{3}{8}$   | B1                     | $\frac{3}{8}$ oe  |  |
|       | $y - 1 = \frac{3}{8}(x - 3)$   | M1                     | Equation of line through either A or B, any non-zero numerical gradient                   |  |
|       | $8y - 8 = 3x - 9$  | A1 3                   | Correct equation in correct form  |  |
|       | $3x - 8y - 1 = 0$  |                        |   |  |
| (ii)  | $\left( \frac{-5+3}{2}, \frac{-2+1}{2} \right)$<br>$= (-1, -\frac{1}{2})$  | M1<br>A1 2             | Uses $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$<br>$(-1, -\frac{1}{2})$    |  |
| (iii) | $AC = \sqrt{(-5+3)^2 + (-2-4)^2}$<br>$= \sqrt{2^2 + 6^2}$<br>$= \sqrt{40}$<br>$= 2\sqrt{10}$   | M1<br>A1<br>A1 3       | Uses $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$<br>$\sqrt{40}$<br>Correctly simplified surd   |  |
| (iv)  | $\text{Gradient of AC} = \frac{-2-4}{-5+3} = 3$<br>$\text{Gradient of BC} = \frac{4-1}{-3-3} = -\frac{1}{2}$<br>$3 \times -\frac{1}{2} \neq -1$ so lines are not perpendicular | B1<br>B1<br>M1<br>A1 4 | 3 oe<br>$-\frac{1}{2}$ oe<br>Attempts to check $m_1 \times m_2$<br>Correct conclusion www |  |
|       |  |                        | <span style="border: 1px solid black; padding: 2px;">12</span>                            |  |

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| 10(i) | $24x^2 - 3x^{-4}$  | B1<br>B1<br>B1 | $24x^2$<br>$kx^{-4}$<br>$-3x^{-4}$   |  |
|       | $48x + 12x^{-5}$   | M1<br>A1 5     | Attempt to differentiate their (i)<br>Fully correct  |  |
| (ii)  | $8x^3 + \frac{1}{x^3} = -9$<br>$8x^6 + 1 = -9x^3$<br>$8x^6 + 9x^3 + 1 = 0$ | *M1            | Use a substitution to obtain a 3-term quadratic  |  |
|       | Let $y = x^3$<br>$8y^2 + 9y + 1 = 0$<br>$(8y + 1)(y + 1) = 0$              | DM1<br>A1      | Correct method to solve quadratic<br>$-\frac{1}{8}, -1$                                    |  |
|       | $y = -\frac{1}{8}, y = -1$<br>$x = -\frac{1}{2}, x = -1$                   | M1<br>A1 5     | Attempt to cube root at least one of their<br>$y$ -values<br>$-\frac{1}{2}, -1$            |  |
|       |  |                | <b>SR</b> one correct $x$ value <b>www</b> <b>B1</b>                                       |  |
|       |  |                | <b>SR for trial and improvement:</b><br>$x = -1$ <b>B1</b><br>$x = -\frac{1}{2}$ <b>B2</b> |  |
|       |  | <b>10</b>      | Justification that there are no further solutions <b>B2</b>                                |  |