

Mark Scheme 4737

January 2006

4737

Mark Scheme

January 2006

<p>1 (i)</p>		<p>M1</p> <p>A1</p>	<p>For $A1$ or $A2$ or both joined to G, P and R $B1$ or $B2$ or both joined to G, T, W and Y and $C1$ or $C2$ or both joined to P, W and Y</p> <p>For both $A1$ and $A2$ joined to G, P and R both $B1$ and $B2$ joined to G, T, W and Y and both $C1$ and $C2$ joined to P, W and Y</p>
<p>(ii)</p>		<p>B1</p>	<p>For $A1$ paired with G and $A2$ with P, or vice versa; $B1$ paired with T and $B2$ with W, or vice versa; one of $C1, C2$ paired with Y and the other left unpaired</p>
<p>(iii)</p>	<p>$R - A2 - P - C2$ or in reverse</p> <p>Accept $R - A - P - C$ or in reverse</p> <p>Amanda gets green and red Ben gets turquoise and white Carrie gets yellow and pink</p>	<p>M1</p> <p>A1</p> <p>B1</p>	<p>For a valid alternating path for their diagram (need not be minimum)</p> <p>For this alternating path</p> <p>For this matching (may use G, R, etc.)</p>
<p>(iv)</p>	<p>Amanda gets pink and red Ben gets green and turquoise Carrie gets white and yellow</p>	<p>B1</p>	<p>For this matching (may use G, R, etc.)</p>

2	Stage	State	Action	Cost	Minimum		
1	0	0	0	3	3		
		1	0	4	4		
2	0	0	0	6+3 = 9	8	M1	For completing cost column for stage 2 For completing cost column for stage 3
		1	0	4+4 = 8			
	1	0	0	6+3 = 9	9	M1	
		1	0	7+4 = 11			
3	0	0	0	5+8 = 13	12	A1	For completing minimum column for stage 2 For completing minimum column for stage 3
		1	0	3+9 = 12			
Shortest route: (3; 0) – (2; 1) – (1; 0) – (0; 0)						B1	For this route (not in reverse)
Length of shortest route = 12 km						B1	

3	(i)	6+3+2+4 = 15 litres per second	B1	For 15
	(ii)	6+2+3+6+2+2 = 21 litres per second Do not use arc <i>DG</i> as it is crossed twice or $X = \{S, A, B, C, E, F\}$ $Y = \{D, G, H, I, T\}$	M1 A1	For showing how 21 (given) was worked out For explaining why arc <i>DG</i> is not used
	(iii)	<i>SC</i> cannot be full since the most that can leave it is 2+4 = 6 litres per second <i>AD</i> cannot be full since the most that can enter it is 2+3 = 5 litres per second The most that can flow in <i>SB</i> is 2+3 = 5 litres per second	B1 B1 B1 B1	For 6 and 'out' or equivalent For 5 and 'in' or equivalent For <i>SB</i> = 5 For 14
	(iv)	Maximum flow = 14 litres per second eg	M1 A1	For a feasible flow (may imply vertex labels) For a feasible flow of 14 litres per second (directions must be shown for the A mark)

	Cut $X = \{S, B, C\}$ $Y = \{A, D, E, F, G, H, I, T\}$ This cut has capacity 14 litres per second		M1 A1	For this cut described or drawn on diagram For explicitly stating that this cut = 14
	Maximum flow \geq this flow = 14 Minimum cut \leq this cut = 14 But maximum flow = minimum cut so 14 is the maximum flow and the minimum cut.		B1	For explaining how maximum flow = minimum cut shows that 14 is the maximum here (at least referring to 'this flow' and 'this cut', not just stating 'max flow = min cut')
				12

4737

Mark Scheme

January 2006

4	(i)	£260	B1	For correct answer with units	
	(ii)	Reduce rows			Or scaled throughout by 10
		0 60 30 10			
		0 90 90 60		M1	For correct method for reducing rows
		0 80 70 40			
		10 10 20 0			
		Reduce columns			
		0 50 10 10		M1	For correct method for reducing columns
		0 80 70 60			
		0 70 50 40		A1	For a correct reduced cost matrix
10 0 0 0					

		Cross through 0's using as few lines as possible	M1	For correct crossing out for their reduced cost matrix. Likely to be shown on reduced cost matrix.	
		0 50 10 10			
		0 80 70 60			
		0 70 50 40			
		10 0 0 0			
		Augment by 10	M1	For correct augmenting from their reduced cost matrix and their crossing out	
		0 40 0 0			
		0 70 60 50			
		0 60 40 30	A1	For a correct solution after first augmenting	
		20 0 0 0			

		Cross through 0's using as few lines as possible	M1	For correct crossing out for their augmented matrix. Likely to be shown on matrix.	
		0 40 0 0			
		0 70 60 50			
		0 60 40 30			
		20 0 0 0			
		Augment by 30	M1	For correctly augmenting their matrix in one step by an amount greater than 10 (or greater than 1 if scaling has been used)	
		30 40 0 0			
		0 40 30 20			
		0 30 10 0	A1	For correct final matrix from completely correct method	
		50 0 0 0			

		Allocation			
		$A = Y$	B1	For this allocation	
		$B = W$			
		$C = Z$	B1	For £180	
		$D = X$			

		Cost = £180			
	(iii)	Hungarian algorithm finds the minimum cost complete matching	B1	For 'minimum cost' or equivalent	

13

4737

Mark Scheme

January 2006

6	(i)	In column Y : $-3 < 5$ so A does not dominate B In column X : $-3 < 2$ (or column Z : $1 < 4$) so B does not dominate A	B1 B1	For $-3 < 5$ or equivalent For $-3 < 2$ or $1 < 4$ or equivalent
	(ii)	The worst outcomes for Maria are: X lose 4, Y lose 5, Z lose 4	M1 A1	For finding <u>column</u> maxima For rejecting 5 as being bigger than 4, or using a word like 'lose' or '-4, -5, -4'
	(iii)	If Lucy plays B she could win as much as 5	B1	For '5 is the most she can win' or equivalent
	(iv)	Need to add 3 throughout matrix to make values non-negative, this removes the 3 again	B1	For 'add 3 throughout matrix' or equivalent
	(v)	Having added 3 throughout, the expected number of points win by Lucy when Maria chooses strategy X is $5p_1 + 0p_2 + 7p_3$, and similarly the second expression is the expected number of points won by Lucy when Maria chooses strategy Y and the third expression is the expected number of points won by Lucy if Maria chooses strategy Z	M1 A1	For showing where one of the expressions came from, or for referring to 'when Maria plays each of her strategies' or equivalent in a non-specific way For specifically linking the expressions to Maria choosing strategy X , strategy Y and strategy Z in that order
	(vi)	The number of points that Lucy can expect to win cannot be less than the worst of the three expressions, so it is less than or equal to each of them.	M1 A1	For reference to 'number of points won by Lucy' or equivalent For reference to 'the worst outcome' or equivalent
	(vii)	$2(p_1) + 4(p_3) = 2p_1 + 4(1-p_1) = 4-2p_1$ (given) $4p_1 - 3(1-p_1) = 7p_1 - 3$	B1 B1	For $2p + 4(1-p)$ For $7p - 3$
	(viii)	$4 - 2p_1 = 7p_1 - 3 \Rightarrow p_1 = \frac{7}{9}$ $p_1 = \frac{7}{9} \Rightarrow 2\frac{4}{9}, p_1 = 0 \Rightarrow \min(4, -3) = -3,$ $p_1 = 1 \Rightarrow \min(2, 4) = 2$ Maximin is when $p_1 = \frac{7}{9} \Rightarrow$ choose randomly between A and C so that A is chosen with probability $\frac{7}{9}$	B1 M1 A1	For solving $4 - 2p_1 =$ their expression to get a probability For evaluating $4-2p_1$ at their p_1 and the values -3 and 2 For reference to maximin, or equivalent, leading to selection of $p = \frac{7}{9}$, or in context
			15	