

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Further Pure 2

MFP2

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Express

$$5 \sinh x + \cosh x$$

in the form $Ae^x + Be^{-x}$, where A and B are integers. (2 marks)

(b) Solve the equation

$$5 \sinh x + \cosh x + 5 = 0$$

giving your answer in the form $\ln a$, where a is a rational number. (4 marks)

2 (a) Given that

$$\frac{1}{r(r+1)(r+2)} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$$

show that $A = \frac{1}{2}$ and find the value of B . (3 marks)

(b) Use the method of differences to find

$$\sum_{r=10}^{98} \frac{1}{r(r+1)(r+2)}$$

giving your answer as a rational number. (4 marks)

3 The cubic equation

$$z^3 + qz + (18 - 12i) = 0$$

where q is a complex number, has roots α , β and γ .

(a) Write down the value of:

(i) $\alpha\beta\gamma$; (1 mark)

(ii) $\alpha + \beta + \gamma$. (1 mark)

(b) Given that $\beta + \gamma = 2$, find the value of:

(i) α ; (1 mark)

(ii) $\beta\gamma$; (2 marks)

(iii) q . (3 marks)

(c) Given that β is of the form ki , where k is real, find β and γ . (4 marks)

4 (a) A circle C in the Argand diagram has equation

$$|z + 5 - i| = \sqrt{2}$$

Write down its radius and the complex number representing its centre. (2 marks)

(b) A half-line L in the Argand diagram has equation

$$\arg(z + 2i) = \frac{3\pi}{4}$$

Show that $z_1 = -4 + 2i$ lies on L . (2 marks)

(c) (i) Show that $z_1 = -4 + 2i$ also lies on C . (1 mark)

(ii) Hence show that L touches C . (3 marks)

(iii) Sketch L and C on one Argand diagram. (2 marks)

(d) The complex number z_2 lies on C and is such that $\arg(z_2 + 2i)$ has as great a value as possible.

Indicate the position of z_2 on your sketch. (2 marks)

Turn over ►

5 (a) Use the definition $\cosh x = \frac{1}{2}(e^x + e^{-x})$ to show that $\cosh 2x = 2 \cosh^2 x - 1$. (2 marks)

(b) (i) The arc of the curve $y = \cosh x$ between $x = 0$ and $x = \ln a$ is rotated through 2π radians about the x -axis. Show that S , the surface area generated, is given by

$$S = 2\pi \int_0^{\ln a} \cosh^2 x \, dx \quad (3 \text{ marks})$$

(ii) Hence show that

$$S = \pi \left(\ln a + \frac{a^4 - 1}{4a^2} \right) \quad (5 \text{ marks})$$

6 By using the substitution $u = x - 2$, or otherwise, find the exact value of

$$\int_{-1}^5 \frac{dx}{\sqrt{32 + 4x - x^2}} \quad (5 \text{ marks})$$

7 (a) Explain why $n(n + 1)$ is a multiple of 2 when n is an integer. (1 mark)

(b) (i) Given that

$$f(n) = n(n^2 + 5)$$

show that $f(k + 1) - f(k)$, where k is a positive integer, is a multiple of 6. (4 marks)

(ii) Prove by induction that $f(n)$ is a multiple of 6 for all integers $n \geq 1$. (4 marks)

8 (a) (i) Expand

$$\left(z + \frac{1}{z}\right)\left(z - \frac{1}{z}\right) \quad (1 \text{ mark})$$

(ii) Hence, or otherwise, expand

$$\left(z + \frac{1}{z}\right)^4 \left(z - \frac{1}{z}\right)^2 \quad (3 \text{ marks})$$

(b) (i) Use De Moivre's theorem to show that if $z = \cos \theta + i \sin \theta$ then

$$z^n + \frac{1}{z^n} = 2 \cos n\theta \quad (3 \text{ marks})$$

(ii) Write down a corresponding result for $z^n - \frac{1}{z^n}$. (1 mark)

(c) Hence express $\cos^4 \theta \sin^2 \theta$ in the form

$$A \cos 6\theta + B \cos 4\theta + C \cos 2\theta + D$$

where A, B, C and D are rational numbers. (4 marks)

(d) Find $\int \cos^4 \theta \sin^2 \theta \, d\theta$. (2 marks)

END OF QUESTIONS

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