Mark Scheme 4724
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1		Attempt to factorise numerator and denominator	M1		
		num = $xx(x-3)$ or denom = $(x-3)(x+3)$	A1		Not num = $x(x^2-3x)$
		Final answer = $\dfrac{x^2}{x+3}$ [Not $\dfrac{xx}{x+3}$]	A1	**3**	Do not ignore further cancellation.
2		$\dfrac{d}{dx}(\sin y) = \cos y.\dfrac{dy}{dx}$	B1		
		$\dfrac{d}{dx}(xy) = x\dfrac{dy}{dx} + y$ s.o.i.	B1		[SR: If xy taken to LHS, accept $-x\dfrac{dy}{dx} + y$ as s.o.i.]
		$\cos y.\dfrac{dy}{dx} = x\dfrac{dy}{dx} + y + 2x$ AEF	B1		
		[If written as $\frac{dy}{dx} = \cos y\frac{dy}{dx} = x\frac{dy}{dx} + y + 2x$, accept for prev B1 but not for following marks if the $\frac{dy}{dx}$ is used]			
		$f(x,y)\dfrac{dy}{dx} = g(x,y)$	M1		Regrouping provided > one $\dfrac{dy}{dx}$ term
		$\dfrac{y+2x}{\cos y - x}$ or $-\dfrac{y+2x}{x-\cos y}$ or $\dfrac{-2x-y}{x-\cos y}$	A1	**5**	ISW Answer could imply M1
3	**(i)**	Quotient = $3x + \ldots$	B1		For correct leading term in quotient
		For evidence of correct division process	M1		Or for cubic $\equiv (x^2-2x+5)(gx+h)\ (+\ldots)$
		$3x+4$	A1		For correct quotient
		$-6x-13$	A1	**4**	For correct remainder ISW
	(ii)	$a = 7$	B1√		Follow through If rem in (i) is $Px+Q$, then B1√ for $a = 1-P$
		$b = 20$	B1√	**2**	and B1√ for $b = 7-Q$
		[SR: If B0+B0, award B1√ for $a = 1+P$ AND $b = 7+Q$; also SR B1 for $a = 20, b = 7$]			
4	**(i)**	Parts using correct split of $u = x, \dfrac{dv}{dx} = \sec^2 x$	M1		1st stage result of form $f(x) +/- \int g(x)\,dx$
		$x\tan x - \int \tan x\,dx$	A1		Correct 1st stage
		$\int \tan x\,dx = -\ln\cos x$ or $\ln\sec x$	B1		
		$x\tan x + \ln\cos x + c$ or $x\tan x - \ln\sec x + c$	A1	**4**	
	(ii)	$\tan^2 x = +/- \sec^2 x +/- 1$	M1		or $\sec^2 x = +/- 1 +/- \tan^2 x$
		$\int x\sec^2 x\,dx - \int x\,dx$ s.o.i.	A1		Correct 1st stage
		$x\tan x + \ln\cos x - \frac{1}{2}x^2 + c$	A1√	**3**	f.t. their answer to part $(i) - \frac{1}{2}x^2$

5 **(i)** $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ M1 Used, not just quoted

$\frac{1}{t}$ or t^{-1} A1 **2** Not $\frac{2}{2t}$ as final answer

SR: M1 for Cart conv, finding $\frac{dy}{dx}$ & ans involv t + A1 M1 is attempt only, accuracy not involved

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(ii) Finding equation of tangent (using p or t) M1

$py = x + p^2$ A1 **2** **AG**; p essential; at least 1 line inter working

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(iii) $(25, -10) \Rightarrow p = -5$ or $-5y = x + 25$ seen B1 $5y = x + 25$ seen $\Rightarrow$ B0

Substitution of their values of p into given tgt eqn M1 Producing 2 equations

Solving the 2 equations simultaneously M1

$(-15, -2)$ $x = -15, y = -2$ A1 **4** Common wrong ans $(15, 8) \Rightarrow$ B0, M2, A0

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6 **(i)** Attempt to connect $dx, d\theta$ M1 But not $dx = d\theta$

$dx = 2\sin\theta\cos\theta\, d\theta$ A1 AEF

$\sqrt{\frac{x}{1-x}} = \frac{\sin\theta}{\cos\theta}$ B1 Ignore any references to $\pm$.

Reduction to $\int 2\sin^2\theta\, d\theta$ A1 **4** **AG** WWW

(ii) $\sin^2\theta = k(+/-1 +/- \cos 2\theta)$ M1 Attempt to change $(2)\sin^2\theta$ into $f(\cos 2\theta)$

$2\sin^2\theta = 1 - \cos 2\theta$ A1 Correct attempt

$\int \cos 2\theta\, d\theta = \frac{1}{2}\sin 2\theta$ B1 Seen anywhere in this part

Attempting to change limits M1 Or Attempting to resubstitute; Accept degrees

$\frac{1}{2}\pi$ A1 **5**

Alternatively Parts once & use

$\cos^2\theta = 1 - \sin^2\theta$ (M2) Instead of the M1 A1 B1

$\frac{1}{2}(\theta - \sin\theta\cos\theta)$ (A1) Then the final M1 A1 for use of limits

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7 **(i)** $A = 3$ B1 For correct value stated

$C = 1$ B1 For correct value stated

$11 + 8x \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$ M1 AEF; any suitable identity

e.g. $A - B = 0, 2A + B - C = 8, A + 2B + 2C = 11$ A1 For any correct (f.t.) equation involving B

$B = 3$ A1 **5**

(ii) $(1 - \frac{x}{2})^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots$ B1 s.o.i.

$(1+x)^{-1} = 1 - x + x^2 - \dots$ B1 s.o.i.

$(1+x)^{-2} = 1 - 2x + 3x^2 - \dots$ B1,B1 s.o.i.

Expansion = $\frac{11}{2} - \frac{17}{4}x + \frac{51}{8}x^2 + \dots$ B1 **5** CAO. No f.t. for wrong A and/or B and/or C

SR(1) If partial fractions not used but product of but $(11+8x)(2-x)^{-1}(1+x)^{-2}$ attempted, then

B1 for $\left(1-\frac{x}{2}\right)^{-1}=1+\frac{x}{2}+\frac{x^2}{4}+\ldots$

B1,B1 for $(1+x)^{-2}=1-2x+\ldots+3x^2+\ldots$

B1,B1 for $\frac{11}{2}-\frac{17}{4}x+\ldots+\frac{51}{8}x^2+\ldots$

SR(2) If partial fractions not used denominator multiplied out, then

B1 for denom $=\ 2+3x\left(+0x^2\right)+\ldots$

B1 for $\left(1+\frac{3x}{2}\right)^{-1}=1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots$

B1,B1,B1 for $\frac{11}{2}\ldots-\frac{17}{4}x\ldots+\frac{51}{8}x^2+\ldots$

N.B. In both SR, if final expansion given B0, -------------------*allow SR B1 for* $22-17x+51/2\,x^2$

8	**(i)**	$\int (y-3)\,\mathrm{d}y=\int (2-x)\,\mathrm{d}x$ or equiv	M1		For separation & integration of both sides
		$\frac{1}{2}y^2-3y=2x-\frac{1}{2}x^2$	A1		or $\frac{1}{2}(y-3)^2=-\frac{1}{2}(x-2)^2$
		For an arbitrary const on one/both sides	*B1		} (or + M2 for equiv statement using limits)
		Substituting $(x,y)=(5,4)$ or (4,5) & finding 'c'	dep*M1		}
		$\frac{1}{2}y^2-3y=-\frac{1}{2}x^2+2x-\frac{3}{2}$ AEF ISW	A1	**5**	or $\frac{1}{2}(y-3)^2=-\frac{1}{2}(x-2)^2+5$ AEF
	(ii)	Attempt to clear fracts (if nec) & compl square	M1		
		$a=2, b=3, k=10$	A2	**3**	For all 3; SR: A1 for 1 or 2 correct
	(iii)	Circle clearly indicated in a sketch	B1		
		Centre $(2,3)$ or their (a,b)	B1√		
		Radius $\sqrt{10}$ or their $\sqrt{k}$	B1√	**3**	√ provided $k>0$
9	**(i)**	Using $\begin{pmatrix}-8\\1\\-2\end{pmatrix}$ and $\begin{pmatrix}-9\\2\\-5\end{pmatrix}$ as the relevant vectors	M1		i.e. correct direction vectors
		Using $\cos\theta=\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$ AEF for any 2 vectors	M1		Accept $\cos\theta=\left\lvert\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}\right\rvert$
		Method for scalar product of any 2 vectors	M1		
		Method for finding magnitude of any vector	M1		
		15° (15.38…), 0.268 rad	A1	**5**	
	(ii)	Produce (at least) 2 of the 3 eqns in t and s	M1		e.g. $4-8t=-2-9s$, $-6-2t=-2-5s$
		Solve the (x) and (z) equations	M1		
		$t=3$ or $s=2$	A1		for first value found
		$s=2$ or $t=3$ f.t.	A1√		for second value found
		Substituting their (t,s) into (y) equation	M1		
		$a=1$	A1		
		Substituting their t into l_1 or their (s,a)			

into l_2	M1		
$\begin{pmatrix} -20 \\ 5 \\ -12 \end{pmatrix}$	A1	**8**	Any format but not $\begin{pmatrix} \\ \\ \\ \end{pmatrix} + \begin{pmatrix} \\ \\ \\ \end{pmatrix}$