4725 Further Pure Mathematics 1

1	**(i)**	$\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	B1		Two elements correct
			B1	**2**	Remaining elements correct
	(ii)	$4a-6$	B1		Correct determinant
			M1		Equate det A to 0 and solve
		$a=\frac{3}{2}$	A1	**3**	Obtain correct answer a. e. f.
			$\boxed{5}$		
2	**(i)**	u^3-3u^2+3u-1	B1		Correct unsimplified expansion of $(u-1)^3$
			M1		Substitute for x
		$2u^3-6u^2+9u-8=0$	A1	**3**	Obtain correct **equation**
	(ii)		M1		Use $(\pm)\frac{\text{d}}{\text{a}}$ of new equation
		4	A1ft	**2**	Obtain correct answer from their equation
			$\boxed{5}$		
3		$x-\text{i}y$	B1		Conjugate known
			M1		Equate real and imaginary parts
		$x+2y=12 \quad 2x+y=9$	A1		Obtain both equations, OK with factor of i
			M1		Solve pair of equations
		$z = 2+5\text{i}$	A1	**5**	Obtain correct answer as a complex number **S.C.** Solving $z + 2\text{i}z = 12 + 9\text{i}$ can get max 4/5, not first B1
			$\boxed{5}$		
4			M1		Express as sum of three series
			M1		Use standard results
		$\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)-n(n+1)$	A1		Obtain correct unsimplified answer
			M1		Attempt to factorise
			A1		Obtain at least factor of $n(n+1)$
		$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	**6**	Obtain fully factorised correct answer
			$\boxed{6}$		

		Mark		Guidance
5	**(i)**	B1		Rotation 90^{o} (about origin)
		B1	**2**	Anticlockwise
	(ii) ***Either***	M1		Show image of unit square after reflection in $y = -x$
	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A1		Deduce reflection in x-axis
		B1ft		Each column correct
		B1ft	**4**	ft for matrix of their transformation
	Or	M1		Post multiply by correct reflection matrix
		A1		Obtain correct answer
		B1B1		State reflection, in x-axis
			S.C.	If pre-multiplication, M0 but B1 B1 Available for correct description of their matrix
		$\boxed{6}$		
6	**(i)**	B1		State or use 5 + i as a root
		M1		Use $\sum \alpha\beta = 6$
	$x = -2$	A1	**3**	Obtain correct answer
	(ii) ***Either***	M1		Use $p = -\sum \alpha$
	$p = -8$	A1ft		Obtain correct answer, from their root
		M1		Use $q = -\alpha\beta\gamma$
	$q = 52$	A1ft	**4**	Obtain correct answer, from their root
	Or	M1		Attempt to find quadratic factor
		M1		Attempt to expand quadratic and linear
		A1A1		Obtain correct answers
	Or	M1		Substitute (5 – i)into equation
		M1		Equate real and imaginary parts
		A1		Obtain correct answer for p
		A1ft		Obtain correct answer for q, ft their p
		$\boxed{7}$		
7	**(i)**	B1	**1**	Obtain **given** answer correctly
	(ii)	M1		Express at least 1st two and last term using **(i)**
		A1		All terms correct
		M1		Show that correct terms cancel
	$1-\frac{1}{(n+1)^2}$	A1	**4**	Obtain correct answer, in terms of n
	(iii) $\frac{1}{4}$	B1		Sum to infinity seen or implied
		B1	**2**	Obtain correct answer
				S.C. -¾ scores B1
		$\boxed{7}$		

8 **(i)**		M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ & $5 - 12i$
	$x^2 - y^2 = 5$ and $xy = -6$	A1		Obtain both results, a.e.f
		M1		Obtain quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)3$ or $y = (\pm)2$
	$\pm(3 - 2i)$	A1	**5**	Obtain correct answers as complex nos
(ii)		B1ft		Circle with centre at their square root
		B1		Circle passing through origin
		B1ft		2^{nd} circle centre correct relative to 1^{st}
		B1	**4**	Circle passing through origin
		$\boxed{9}$		
9 **(i)**		M1		Show correct expansion process for 3×3 or multiply adjoint by **A**
		M1		Correct evaluation of any 2×2 at any stage
	det **A** = $\Delta = 6a - 6$	A1		Obtain correct answer
	$\mathbf{A}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	M1		Show correct process for adjoint entries
		A1		Obtain at least 4 correct entries in adjoint
		B1		Divide by their determinant
		A1	**7**	Obtain completely correct answer
(ii)	$\frac{1}{\Delta}\begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$	M1		Attempt product of form $\mathbf{A}^{-1}\mathbf{C}$ or eliminate to get 2 equations and solve
		A1A1A1 ft all 3		Obtain correct answer
			4	**S.C.** if det now omitted, allow max A2 ft
		$\boxed{11}$		
10 (i)		B1		Correct $\mathbf{M}^2$ seen
	$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ $\mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	M1		Convincing attempt at matrix multiplication for $\mathbf{M}^3$
		A1	**3**	Obtain correct answer
(ii)	$\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$	B1ft	**1**	State correct form, consistent with **(i)**

10 (iii)	M1		Correct attempt to multiply **M** & $\mathbf{M}^k$ or v.v.
	A1		Obtain element 2(k + 1)
	A1		Clear statement of induction step, from correct working
	B1	**4**	Clear statement of induction conclusion, following their working
(iv)	B1		Shear
	DB1		x-axis invariant
	DB1	**3**	e.g. (1, 1) $\rightarrow$ (21, 1) or equivalent using scale factor or angles
	$\boxed{\mathbf{11}}$		