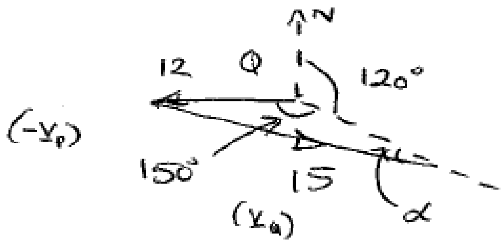
June 2006
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
1.		M1
	$\dfrac{\sin\alpha}{12} = \dfrac{\sin 150^\circ}{15}$	M1 A1
	$\Rightarrow \sin\alpha = \dfrac{6}{15}$	
	$\Rightarrow \alpha = 23.6^\circ$	A1
	$\therefore$ Course is 096 (.4°)	A1 (5)
2.		
	$(\rightarrow)$ $u\cos\alpha = v\cos\theta$	M1 A1
	$(\uparrow)$ $eu\sin\alpha = v\sin\theta$	M1 A1
	$\Rightarrow v^2 = u^2(\cos^2\alpha + e^2\sin^2\alpha)$	M1
	$\Rightarrow KE = \frac{1}{2}mu^2(\cos^2\alpha + e^2\sin^2\alpha)$	A1 (6)

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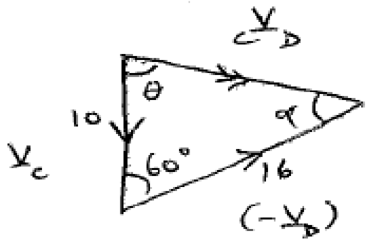
3. (a)	 $|{}_C\underline{v}_D|^2 = 10^2 + 16^2 - 2\times10\times16\cos60^\circ$ $= 196$	M1 A1
	$\underline{|{}_C\underline{v}_D| = 14 \text{ ms}^{-1}}$ *	A1 (3)
(b)	α is <u>acute</u> (opposite shortest side)	
	$\frac{\sin\alpha}{10} = \frac{\sin 60^\circ}{14}$	M1
	$\Rightarrow \alpha = 38.213^\circ..$	A1
	 (i) $DN = 4000\sin 8.213$	M1
	$\simeq \underline{571\text{ m}}$ $\left(\frac{4000}{7}\right)$	A1
	(ii) $t = \frac{4000\cos 8.213^\circ}{14}$ secs.	M1 A1
	$\simeq 282.78...$ secs.	
	<u>Time is 2.05 pm (nearest minute)</u>	A1 (7)
		(10)

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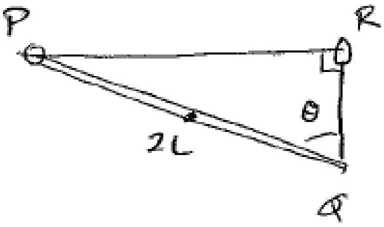
4.(a)		
	PE of rod $= -mgl\cos\theta$	B1
	EPE of string $= \frac{kmg}{2l}(2l\cos\theta - l)^2$	M1 A1
	Total PE of system, $V = -mgl\cos\theta + \frac{kmgl}{2}(2\cos\theta - 1)^2 + c$	M1
	$= -mgl\cos\theta + \frac{kmgl}{2}(4\cos^2\theta - 4\cos\theta + 1) + c$	M1 A1
	$= mgl(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= mgl[2k\cos^2\theta - (2k+1)\cos\theta] + c'$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgl(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^m, $mgl\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta > 0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\frac{1}{2} < k$ *	A1 (5)
		(12)

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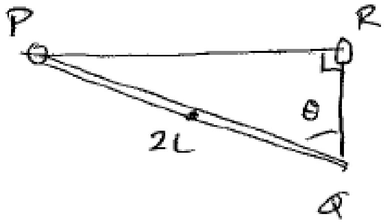
4.(a)	 PE of rod $= -mgL\cos\theta$	B1
	EPE of string $= \frac{kmg}{2L}(2L\cos\theta - L)^2$	M1 A1
	Total PE of system, $V = -mgL\cos\theta + \frac{kmgL}{2}(2\cos\theta-1)^2 + c$	M1
	$= -mgL\cos\theta + \frac{kmgL}{2}(4\cos^2\theta - 4\cos\theta + 1) + c$	M1 A1
	$= mgL(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= mgL[2k\cos^2\theta - (2k+1)\cos\theta] + c'$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgL(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^m, $mgL\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta > 0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\frac{1}{2} < k$ *	A1 (5)
		(12)

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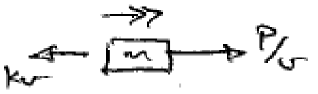
5. (a)	 $\frac{P}{v}$	B1
	$(\rightarrow):\quad \frac{P}{v} - kv = m\frac{dv}{dt}$	M1
	$\Rightarrow \underline{P = mv\frac{dv}{dt} + kv^2}$ *	A1 (3)
(b)	$\int_0^T dt = \int_U^{2U} \frac{mv\,dv}{P - kv^2} \qquad \left(U = \frac{1}{3}\sqrt{\frac{P}{k}}\right)$	M1 A1
	$\Rightarrow \quad T = \frac{-m}{2k}\left[\ln(P - kv^2)\right]_U^{2U}$	A2
	$= \frac{m}{2k}\left\{\ln\left(P - \frac{k}{9}\cdot\frac{P}{k}\right) - \ln\left(P - \frac{4k}{9}\cdot\frac{P}{k}\right)\right\}$	M1 A1
	$= \frac{m}{2k}\left\{\ln\frac{8P}{9} - \ln\frac{5P}{9}\right\}$	
	$= \frac{m}{2k}\ln\left(\frac{8P}{9}\times\frac{9}{5P}\right)$	M1
	$= \underline{\frac{m}{2k}\ln\frac{8}{5}}$	A1 (8)
		(11)

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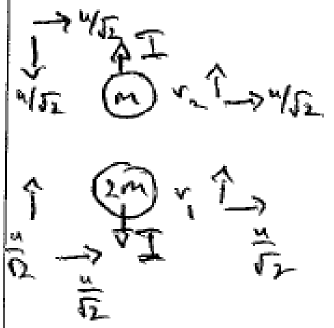
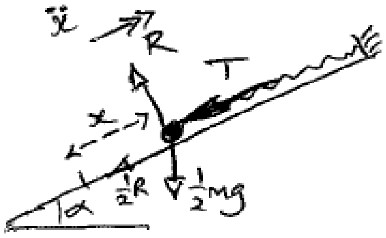
6. (a)		
	Form: $I = m\left(v_2 + \frac{u}{\sqrt{2}}\right)$	M1 A1
	CLM(↑): $\frac{2mu}{\sqrt{2}} - \frac{mu}{\sqrt{2}} = 2mv_1 + mv_2$	
	$\frac{u}{\sqrt{2}} = 2v_1 + v_2$ — ①	M1 A1
	NIL: $e\frac{2u}{\sqrt{2}} = \frac{u}{\sqrt{2}} = -v_1 + v_2$ — ②	M1 A1
	$\Rightarrow \frac{3u}{\sqrt{2}} = 3v_2$ (3s cancelled)	M1 A1
	$\Rightarrow I = m\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)$	
	$= mu\sqrt{2}$	A1 (9)
(b)	$v_2 - v_1 = \frac{u}{\sqrt{2}}$ (Separation speed)	M1
	time to wall $= \frac{d}{u/\sqrt{2}} = \frac{d\sqrt{2}}{u}$	M1 A1
	∴ Separation $= \frac{d\sqrt{2}}{u} \times \frac{u}{\sqrt{2}} = d$	M1 A1
		(5)
		(14)

Question number	Scheme	Marks
7.(a)		
	$F=\frac{1}{2}R$	M1
	$R=mg\cos\alpha$	B1
	$T=\frac{4mgx}{L}$	B1
	$(\rightarrow):\quad -F-mg\sin\alpha-T=m\ddot{x}$	M1 A1
	$-\frac{1}{2}\cdot\frac{4}{5}mg-\frac{3}{5}mg-\frac{4mgx}{L}=m\ddot{x}$	
	$\Rightarrow \frac{d^2x}{dt^2}+4\omega^2x=-g$ *	A1
	$(\omega=\sqrt{g/L})$	(6)
(b)	$m^2+4\omega^2=0 \Rightarrow m=\pm 2\omega i$	
	C.F. is $x=A\sin 2\omega t+B\cos 2\omega t$	M1
	P.I. is $x=\frac{-g}{4\omega^2}=-L/4$	B1
	G.S. is $x=A\sin 2\omega t+B\cos 2\omega t-\frac{L}{4}$	B1
	$t=0, x=0$: $B=L/4$	M1 A1
	$\dot{x}=2\omega A\cos 2\omega t-2\omega B\sin 2\omega t$	
	$t=0, \dot{x}=\frac{1}{2}\sqrt{gL}$: $\frac{\sqrt{gL}}{2}=2\sqrt{\frac{g}{L}}A \Rightarrow A=L/4$	M1
	$\Rightarrow x=\frac{L}{4}(\sin 2\omega t+\cos 2\omega t-1)$	A1 (7)
(c)	$\dot{x}=0 \Rightarrow 2\omega A\cos 2\omega t-2\omega B\sin 2\omega t=0$	M1
	$\Rightarrow \tan 2\omega t=\frac{A}{B}=1$	
	$\Rightarrow 2\omega t=\pi/4$ (first value)	A1
	$\Rightarrow x=\frac{L}{4}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-1\right)$	M1
	$=\frac{L}{4}(\sqrt{2}-1)$	A1 (4)
		(17)