

General Certificate of Education
June 2009
Advanced Level Examination



MATHEMATICS
Unit Mechanics 3

MM03

Wednesday 17 June 2009 9.00 am to 10.30 am

For this paper you must have:

- a 12-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A ball of mass m is travelling vertically downwards with speed u when it hits a horizontal floor. The ball bounces vertically upwards to a height h .

It is thought that h depends on m , u , the acceleration due to gravity g , and a dimensionless constant k , such that

$$h = km^\alpha u^\beta g^\gamma$$

where α , β and γ are constants.

By using dimensional analysis, find the values of α , β and γ . (5 marks)

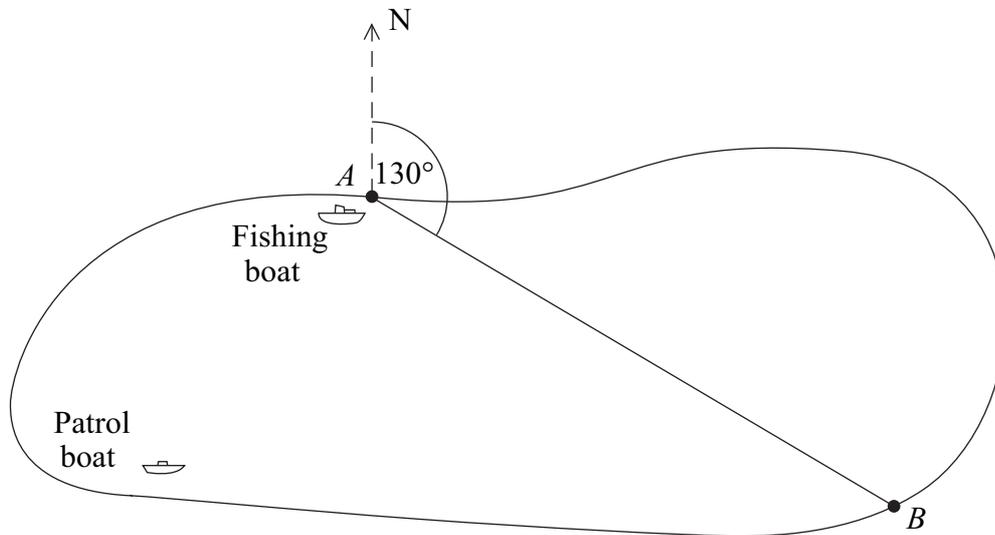
- 2 A particle is projected from a point O on a horizontal plane and has initial velocity components of 2 m s^{-1} and 10 m s^{-1} parallel to and perpendicular to the plane respectively. At time t seconds after projection, the horizontal and upward vertical distances of the particle from the point O are x metres and y metres respectively.

- (a) Show that x and y satisfy the equation

$$y = -\frac{g}{8}x^2 + 5x \quad (4 \text{ marks})$$

- (b) By using the equation in part (a), find the horizontal distance travelled by the particle whilst it is more than 1 metre above the plane. (4 marks)
- (c) Hence find the time for which the particle is more than 1 metre above the plane. (2 marks)

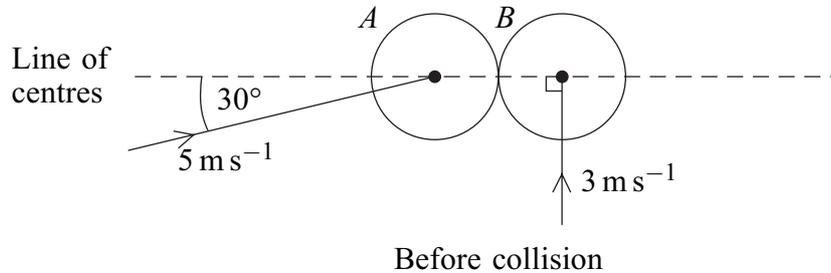
- 3 A fishing boat is travelling between two ports, A and B , on the shore of a lake. The bearing of B from A is 130° . The fishing boat leaves A and travels directly towards B with speed 2 m s^{-1} . A patrol boat on the lake is travelling with speed 4 m s^{-1} on a bearing of 040° .



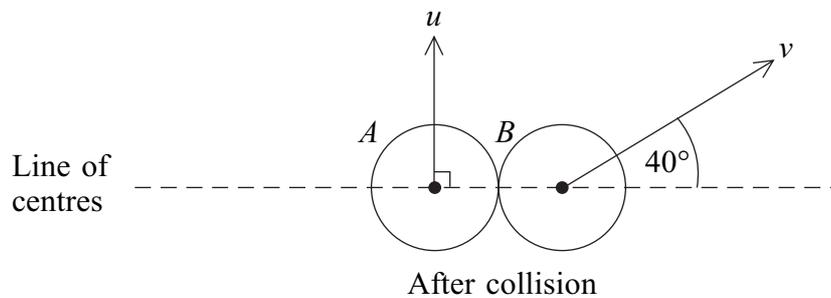
- (a) Find the velocity of the fishing boat relative to the patrol boat, giving your answer as a speed together with a bearing. (5 marks)
- (b) When the patrol boat is 1500 m due west of the fishing boat, it changes direction in order to intercept the fishing boat in the shortest possible time.
- (i) Find the bearing on which the patrol boat should travel in order to intercept the fishing boat. (4 marks)
- (ii) Given that the patrol boat intercepts the fishing boat before it reaches B , find the time, in seconds, that it takes the patrol boat to intercept the fishing boat after changing direction. (4 marks)
- (iii) State a modelling assumption necessary for answering this question, other than the boats being particles. (1 mark)
- 4 A particle of mass 0.5 kg is initially at rest. The particle then moves in a straight line under the action of a single force. This force acts in a constant direction and has magnitude $(t^3 + t) \text{ N}$, where t is the time, in seconds, for which the force has been acting.
- (a) Find the magnitude of the impulse exerted by the force on the particle between the times $t = 0$ and $t = 4$. (3 marks)
- (b) Hence find the speed of the particle when $t = 4$. (2 marks)
- (c) Find the time taken for the particle to reach a speed of 12 m s^{-1} . (5 marks)

- 5 Two smooth spheres, A and B , of equal radii and different masses are moving on a smooth horizontal surface when they collide.

Just before the collision, A is moving with speed 5 m s^{-1} at an angle of 30° to the line of centres of the spheres, and B is moving with speed 3 m s^{-1} perpendicular to the line of centres, as shown in the diagram below.

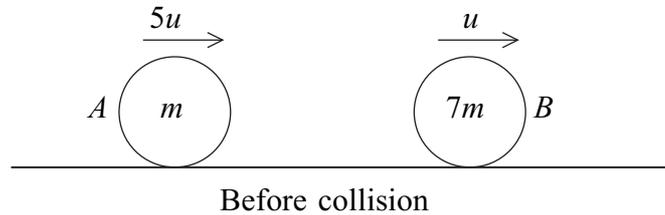


Immediately after the collision, A and B move with speeds u and v in directions which make angles of 90° and 40° respectively with the line of centres, as shown in the diagram below.



- (a) Show that $v = 4.67 \text{ m s}^{-1}$, correct to three significant figures. (3 marks)
- (b) Find the coefficient of restitution between the spheres. (3 marks)
- (c) Given that the mass of A is 0.5 kg , show that the magnitude of the impulse exerted on A during the collision is 2.17 N s , correct to three significant figures. (3 marks)
- (d) Find the mass of B . (3 marks)

- 6 A smooth sphere A of mass m is moving with speed $5u$ in a straight line on a smooth horizontal table. The sphere A collides directly with a smooth sphere B of mass $7m$, having the same radius as A and moving with speed u in the same direction as A . The coefficient of restitution between A and B is e .

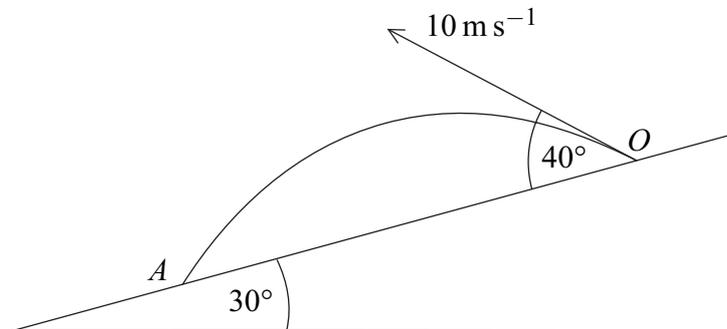


- (a) Show that the speed of B after the collision is $\frac{u}{2}(e + 3)$. (5 marks)
- (b) Given that the direction of motion of A is reversed by the collision, show that $e > \frac{3}{7}$. (4 marks)
- (c) Subsequently, B hits a wall fixed at right angles to the direction of motion of A and B . The coefficient of restitution between B and the wall is $\frac{1}{2}$. Given that after B rebounds from the wall both spheres move in the same direction and collide again, show also that $e < \frac{9}{13}$. (4 marks)

Turn over for the next question

Turn over ►

- 7 A particle is projected from a point O on a smooth plane which is inclined at 30° to the horizontal. The particle is projected down the plane with velocity 10 m s^{-1} at an angle of 40° above the plane and first strikes it at a point A . The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



- (a) Show that the time taken by the particle to travel from O to A is

$$\frac{20 \sin 40^\circ}{g \cos 30^\circ} \quad (3 \text{ marks})$$

- (b) Find the components of the velocity of the particle parallel to and perpendicular to the slope as it hits the slope at A . (4 marks)
- (c) The coefficient of restitution between the slope and the particle is 0.5. Find the speed of the particle as it rebounds from the slope. (4 marks)

END OF QUESTIONS

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