

Question Number	Scheme	Marks
1.	<p>Work done by force = <math>\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \mathbf{AB}</math></p> <p>Attempt at equating work done to KE</p> $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ 2\lambda \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} x-2 \\ y+2 \\ z+3 \end{pmatrix} = \frac{1}{2}(0.1)5^2$ <p>Solving for <math>\lambda</math> (<math>\lambda = 0.25</math>) or forming sufficient equations in <math>x, y</math> [e.g. <math>x + 2y = -0.75</math>, <math>y + 2 = 2(x - 2)</math>]</p> <p>Method to find <b>OB</b></p> $[\mathbf{OB} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \text{ or solving for } x, y]$ <p><b>OB</b> = <math>2\frac{1}{4}\mathbf{i} - 1\frac{1}{2}\mathbf{j} - 3\mathbf{k}</math></p>	M1 M1 A1 M1 M1 any form A1 <b>(6 marks)</b>
Alt.	<p>Non-vector approach:</p> <p><math> \mathbf{F}  \cos \theta = ma</math> applied; <math>[a = 10\sqrt{5}]</math></p> <p>Method to find “s”: <math>5^2 = 2(10\sqrt{5})s</math> <math>[s = \frac{\sqrt{5}}{4}]</math></p> <p>Finding <math>\lambda</math></p> <p>Method to find <b>OB</b></p>	M1 M1 A1 M1 M1 A1

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2. (a)	<p>Integrating factor approach:</p> $IF = e^{\int 1 dt} = e^t$ <p>Multiplying through <math>\Rightarrow \frac{d}{dt} (re^t) = (\mathbf{i} - \mathbf{j}) e^{-t}</math></p> <p>Integrating <math>\Rightarrow r e^t = -(\mathbf{i} - \mathbf{j}) e^{-t} (+ \mathbf{c})</math></p> <p>Using <math>\mathbf{r} = \mathbf{0}</math>, <math>t = 0</math> to find <math>\mathbf{c}</math> <math>[\mathbf{c} = \mathbf{i} - \mathbf{j}]</math></p> $\Rightarrow \mathbf{r} = -(\mathbf{i} - \mathbf{j}) e^{-2t} + (\mathbf{i} - \mathbf{j}) e^{-t}$	B1 M1A1 M1 A1 ft M1 A1 (7)
(b)	<p>Writing <math>\mathbf{r} = f(t)\mathbf{i} + g(t)\mathbf{j}</math> or <math>x = f(t)</math>, <math>y = g(t)</math> and attempt to eliminate <math>t</math></p> $y = -x$	M1 A1 (2) (9 marks)
Alt. (a)	<p>AE <math>m + 1 = 0 \Rightarrow \mathbf{r} = \mathbf{A}e^{-t}</math> [Form of PI: <math>\mathbf{r} = \mathbf{B}e^{-2t}</math>]</p> <p>Equation for PI: <math>-2 e^{-2t} \mathbf{B} + \mathbf{B}e^{-2t} = (\mathbf{i} - \mathbf{j})e^{-2t}</math></p> $\mathbf{B} = -(\mathbf{i} - \mathbf{j})$ <p>General Solution: <math>\mathbf{r} = \mathbf{A}e^{-t} + (-\mathbf{i} + \mathbf{j})e^{-2t}</math></p> <p>Using <math>\mathbf{r} = \mathbf{0}</math>, <math>t = 0</math> to find <math>\mathbf{A}</math></p> $\mathbf{r} = (\mathbf{i} - \mathbf{j})e^{-t} + (-\mathbf{i} + \mathbf{j})e^{-2t}$	B1 M1 A1 A1 ft M1 M1 A1 (7)

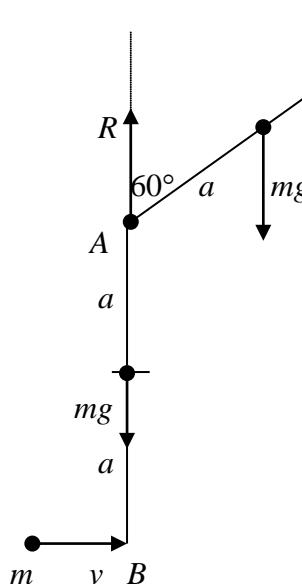
(ft = follow through mark)

Question Number	Scheme	Marks
3. (a)	$\mathbf{R} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 2 \end{pmatrix}$ or $8\mathbf{i} + 2\mathbf{k}$	M1 A1 (2)
(b)	Finding one of $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$ , $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ , $\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ , $\begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 0 \\ 0 \\ -6 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}$	M1 A2, 1, 0
	[A1 one correct, A2 at least three correct]	
	Resultant $= \begin{pmatrix} 4 \\ 0 \\ -16 \end{pmatrix}$ any form	M1 A1 (5)
(c)	$\mathbf{F} = -8\mathbf{i} - 2\mathbf{k}$	B1ft (1)
(d)	For equilibrium $\mathbf{r} \times \begin{pmatrix} -8 \\ 0 \\ -2 \end{pmatrix} = -\begin{pmatrix} 4 \\ 0 \\ -16 \end{pmatrix}$ or equivalent $\mathbf{P}\mathbf{X} = \begin{pmatrix} 0 \\ \lambda \\ 0 \end{pmatrix} \Rightarrow \mathbf{r} \times \begin{pmatrix} -8 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -2\lambda \\ 0 \\ 8\lambda \end{pmatrix}$	M1 M1 A1 ft
	Finding $\lambda$ ; $PX = 2$ .	M1; A1 (5) <b>(13 marks)</b>

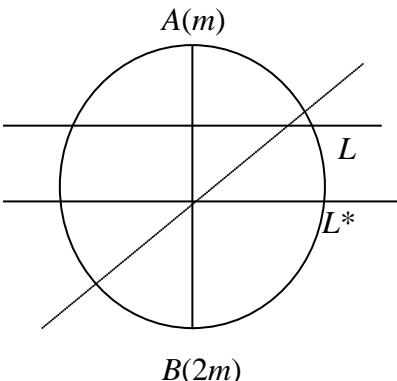
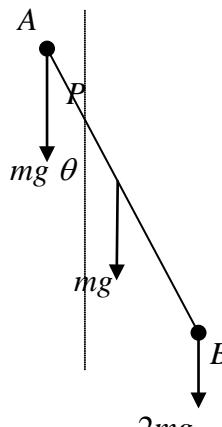
(ft = follow through mark)

Question Number	Scheme	Marks
4. (a)	$(m + \delta m)(v + \delta v) + (-\delta m)(v - U) - mv = -kv \delta t$ $\Rightarrow m \frac{dv}{dt} + U \frac{dm}{dt} = -kv$ $m = M - \lambda t$ $\Rightarrow (M - \lambda t) \frac{dv}{dt} = \lambda U - kv$ $\Rightarrow \frac{dv}{dt} = \frac{\lambda U - kv}{M - \lambda t} \quad (*)$	M1 A1 B1 M1 A1 (no incorrect working seen) A1 cso (7)
(b)	Separating variables: $\int \frac{dv}{U - v} = \int \frac{10}{M - 10t} dt$ Integrating: $\ln(U - v) = \ln(M - 10t) + c$ Using limits correctly: $[ ]_v^0 = [ ]_t^0$ applied or $t = 0, v = 0$ to find "c" Complete method to find $v$	or equivalent M1 M1 A1 M1 $[c = \ln\left(\frac{U}{M}\right)]$ $[\ln\left(\frac{U}{U-v}\right) = \ln\left(\frac{M}{M-10t}\right)]$
	$v = \frac{10Ut}{M}$	A1 (6) <b>(13 marks)</b>

(cso = correct solution only)

Question Number	Scheme	Marks
5. (a)	 <p> <math>I_A = \left\{ \frac{4}{3}ma^2 + m(2a)^2 \right\}</math>  <math>mv(2a) = I_A \omega = \frac{16ma^2}{3}\omega</math>  <math>\omega = \frac{3v}{8a}</math> * no wrong working seen  Gain in PE = <math>mg 3a (1 + \cos 60^\circ)</math>  Attempt at <math>\frac{1}{2}I\omega^2 = \text{gain in PE}</math>  <math>\frac{1}{2} \left( \frac{16ma^2}{3} \right)_c \left( \frac{3v}{8a} \right)^2 = mg 3a (1 + \cos 60^\circ)</math>  Finding <math>v</math>      <math>v = \sqrt{12ga}</math> </p>	M1 A1 M1 A1ft A1 cso (5) M1 A1 M1 A1 ft M1 A1 (6)
(c)	<p>Acceleration of C of G = <math>(\frac{3}{2}a\omega^2)</math></p> <p><math>R - 2mg = mr\omega^2</math>; <math>= 2m(\frac{3}{2}a\omega^2)</math></p> <p>Substitution of <math>\omega</math> and <math>v</math> and finding <math>R = \dots</math></p> <p><math>R = \frac{113}{16}mg</math></p>	B1 M1 A1 M1 A1 (5)
		<b>(16 marks)</b>

(cso = correct solution only; ft = follow through mark)

Question Number	Scheme	Marks
6 (a)	$(\delta I) = (\rho)2\pi r \delta r \times r^2$ Using $(\rho) = \frac{m}{\pi a^2}$ Completion: $I = \frac{2m}{a^2} \left[ \frac{r^4}{4} \right]_0^a = \frac{1}{2} ma^2$ (*)	M1 M1 M1 A1 (4)
	 Disc: Use of $\perp^r$ axis theorem to find $I_{L^*}$ $I_{L^*} = \frac{1}{2} (\frac{1}{2} ma^2) = \frac{1}{4} ma^2$ Use of parallel axis theorem $I_L = \frac{1}{4} ma^2 + m \left( \frac{a}{2} \right)^2 = \frac{1}{2} ma^2$	M1 A1 M1 A1
	For loaded disc: $I = \frac{1}{2} ma^2 + m \left( \frac{a}{2} \right)^2 + 2m \left( \frac{3a}{2} \right)^2 = \frac{21}{4} ma^2$ (*)	M1 A1 cso (6)
(c)	$I \ddot{\theta} = \left\{ mg \left( \frac{a}{2} \right) \sin \theta - mg \left( \frac{a}{2} \right) \sin \theta - 2mg \left( \frac{3a}{2} \right) \sin \theta \right\}$ [A1 for signs, A1 “terms”]  $\left[ \frac{21}{4} ma^2 \ddot{\theta} = -3mga \sin \theta \right]$ For small angles $\theta \approx \sin \theta \Rightarrow$ $\frac{21}{4} ma^2 \ddot{\theta} = -3mga \theta$ $\ddot{\theta} = -\frac{4g}{7a} \theta$ $\Rightarrow \text{ SHM with } \omega^2 = \frac{4g}{7a}$ Time = $\frac{\pi}{\omega}$ ; $= \pi \sqrt{\frac{7a}{4g}}$ or $\frac{\pi}{2} \sqrt{\frac{7a}{g}}$	M1 A1 A1 M1 A1 ft M1 M1; A1 (8)

(18 marks)

(cao = correct answer only; ft = follow through mark; (\*) indicates final line is given on the paper)