4734 Probability & Statistics 3

1	T has a Poisson distribution E(T)=28×0.75+4×6.4 = 46.6 Var(T)=46.6	B1 M1 A1 B1√ **4**	 From sum of Poissons Ft E(T) only if Poisson
2 (i)	Use F(Q_3)=0.75 or $\int_{Q_3}^{\infty} \frac{1}{5} e^{-\frac{1}{4}u}\,\mathrm{d}u = 0.25$ Solve to obtain Q_3 = 4.65 AEF eg 4ln(16/5)	M1 M1A1 **3**	 M1 for solving similar eqn A0 for ≥4.65
(ii)	$\mathrm{f}(u) = \begin{cases} \frac{1}{5}\mathrm{e}^{u} & u < 0, \\ \frac{1}{5}\mathrm{e}^{-\frac{1}{4}u} & u \geq 0. \end{cases}$	B1 B1 **2**	u < 0 unless evidence of ∫ u ≥ 0
3 (i)	Use 28± zs z=2.326 s^2 = 28×72/1200 (25.0, 31.0)	M1 B1 B1 A1 **4**	Accept s=c/√n for M1 Accept 0.28 with corresponding s Or 1199 Accept (25, 31)
(ii)	2×2.326 √(0.28×0.72/n) ≤0.05 AEF Solve to obtain n Smallest n = 1745 e.g. Variance is an approximation	M1 M1 A1 B1 **4**	Or = or ≥ Solving similar equn Accept 1746 ,1750 Or normal is approx or Or p only an estimate
4 (i)	c = 1/20	B1 **1**	
(ii)	$\int_{25}^{45} \frac{400\sqrt{x} - 240}{20}\,\mathrm{d}x$ $= \left[\frac{40}{3}x^{3/2} - 12x\right]$ = 2118(£)	M1 A1 A`1 **3**	 Correct indefinite integral 2120 or better than 2118
(iii)	400√X –240 > 2000 , X > 31.36 P(X > 31.36) = (45 – 31.36)/20 = 0.682	M1 M1 A1 **3**	Or 31.4 cao

5 (i)	$H_o: \mu_2 = \mu_1$, $H_1: \mu_2 > \mu_1$, where μ_1 and μ_2 are the mean concentrations in the lake before and after the spillage respectively	B1 B1 **2**	For both hypotheses Allow in words if population mean used**.**
(ii)	$\bar{X}_2 - \bar{X}_1 \geq zs$ z=1.645 s=0.24√(1/5+1/6) ≥ 0.2391	M1 A1 B1 A1 **4**	Accept >, =, <. ≤, *ts* Or >**;** 0.239
(iii)	$P(\bar{X}_2 - \bar{X}_1 < 0.2391)$ *z* =[0.2391 – 0.3]/*s* *p*=0.3376 This is a large probability for this error	M1 M1 A1 B1 **4**	May be implied ART 0.337 or 0.338 Relevant comment
6 (i)	Use *B* ~ B(29, 0.3) , *G* ~ B(26, 0.2) E(*F*)=29×0.3+26×0.2=13.9 Var(*F*) = 29×0.3×0.7+26×0.2×0.8=10.25	M1 M1A1 M1A1 **5**	
(ii)	*B: np* = 8.7, *nq*=20.3 *G*: *np* = 5.2, *nq*= 20.8 All exceed 5, so normal approximation valid for each *F* ~ N(13.9, 10.25) (approximately) (Requires P(*F* ≤*n*) = 0.99) [*n* + 0.5 – 13.9]/√(10.25) ; = 2.326, their 10.25 *n* = 20.85 Need to have 21 spares available SR Using B(55, 0.2527): B1; M1(N(13.9, 10.39); M1B1M1A0 (Max 5/8)	 B2 M1√ M1B1 A1 M1 A1 **8**	Must check numerically B1 for checking one distribution Use normal. May be implied Standardise M0 if variance has divisors cc Solving similar No cc, lose last A1 (n = 22) Wrong cc, lose A1A1

7 (i)	Requires population of (2nd mark – 1st mark) to be normally distributed H_o: $\mu_d = 0$, H_1:$\mu_d > 0$ $T_2 - T_1$: - 1 - 1 2 0 - 2 2 3 2 $\bar{d} = 0.625$, $s^2 = 3.411$ ($3^{23}/_{56}$ or $^{191}/_{56}$) Use 2.998 EITHER: $t = 0.625/\sqrt{(3.411/8)}$ $= 0.957$ OR: CV(CR), $\bar{d} \geq 2.998\sqrt{3.411/8}$ $= 1.958$ EITHER 0.957<2.998 OR 0.625 < 1.958 Do not reject H_o, there is insufficient evidence of improvement	B1 M1 B1B1 B1 M1 A1 M1 A1 M1 **8**	M0 if clearly z With comparison and conclusion
(ii)	Use $E(X_2 - X_1 + k) = 0.625 + k$ Requires $(0.625+k) / \sqrt{(3.411/8)} \geq 2.998$ Giving $k \geq 1.33$ Increase each mark by 2	M1 A1√ A1 **3**	Allow 1.33
8 (i)	Mean= (20+16+9)/75 = 0.6 $3p = 0.6$, $p= 0.2$ AG	M1 A1 A1 **3**	
(ii)	H_0: B(3,p) fits the data (H_1: B(3,p) does not fit the data) Expected values 38.4 28.8 7.2 0.6 Combine last two cells $\chi^2 = 5.6^2/38.4+ 8.8^2/28.8+3.2^2/7.8$ $= 4.818$ 4.818 > 3.841 Reject H_0 and conclude that there is insufficient evidence that B(3,p) fits the data.	B1 M1 A1 A1 B1 M1 A1√ A1 B1√ M1 **10**	Or: X~B(3,p) or B(3,0.2) Not 'Data fits model' Use B(3,0.2)×75 At least 2 correct All correct With one correct At least 2 correct Ft E values Accept 4.82 cao ft 4.818 SR1 If cells not combined: B1M1A1A1B0M1A1A0B1(5.991)M1 SR2:E-values rounded :B1M1A1A1 B1M1A1A0(4.865)B1M1
(iii)	2.74 < 3.841, accept H_0 conclude that B(6, p) fits the data	B1 **1**	Accept with no reason if evidence of method in (ii)