

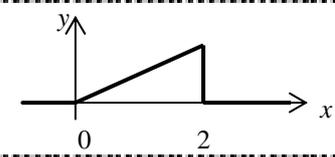
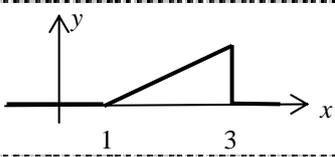
4733 Probability & Statistics 2

1	$\frac{80 - \mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$ $\frac{\mu - 50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$ Solve simultaneously $\mu = 58.7, \sigma = 12.9$	M1 B1 A1 M1 A1 A1	6	Standardise once with Φ^{-1} , allow σ^2 , cc Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs Both equations correct apart from wrong z , <i>not</i> 1–1.645 Solve two standardised equations μ , a.r.t 58.7 σ , a.r.t. 12.9 [<i>not</i> σ^2] [σ^2 : M1B1A0M1A1A0]
2 (i)	Let R denote the number of choices which are 500 or less. $R \sim B(12, \frac{5}{6})$ $P(R = 12) = (\frac{5}{6})^{12} [=0.11216]$ = 0.112	M1 M1 A1	3	$B(12, \frac{5}{6})$ stated or implied, allow 501/600 etc p^{12} or q^{12} or equivalent Answer, a.r.t. 0.112 [SR: $\frac{500}{600} \times \frac{499}{599} \times \frac{498}{598} \times \dots$; 0.110: M1A1] [M1 for 0.910 or 0.1321 or vague number of terms]
(ii)	Method unbiased; unrepresentative by chance	B1 B1	2	State that method is unbiased Appropriate comment (e.g. “not unlikely”) [SR: partial answer, e.g. not <u>necessarily</u> biased: B1]
3 (i)	$P(\leq 1) = 0.0611$ $P(\geq 9) = 1 - P(\leq 8) = 1 - 0.9597$ = 0.0403 $0.0611 + 0.0403 [= 0.1014]$ = 10.1%	B1 M1 A1 M1 A1	5	0.0611 seen Find $P(\geq 9)$, allow 8 or 10 [0.0866, 0.0171] 0.0403 correct Add probabilities of tails, <i>or</i> 1 tail \times 2 Answer [10.1, 10.2]% <i>or</i> probability
(ii)	$P(2 \leq G \leq 8)$ = 0.8944 – 0.0266 [= 0.8678] = 0.868	M1 M1 A1	3	Attempt at $P(2 \leq G \leq 8)$, <i>not</i> isw, allow $1 \leq G \leq 9$ etc Po(5.5) tables, $P(\leq \text{top end}) - P(\leq \text{bottom end})$ Answer, a.r.t. 0.868, allow %
4 (i)	$\hat{\mu} = \bar{y} = \frac{3296.0}{40} = 82.4$ $\frac{286800.4}{40} - 82.4^2 [= 380.25]$ $S^2 \times \frac{40}{39} ; = 390$	B1 M1 M1 A1	4	Mean 82.4, c.a.o. Use correct formula for biased estimate Multiply by $n/(n - 1)$ [SR: all in one, M2 or M0]
(ii)	$\Phi\left(\frac{60 - 82.4}{\sqrt{390}}\right) = \Phi(-1.134)$ = $1 - 0.8716 =$ 0.128	M1 A1	2	Standardise, allow 390, cc or biased estimate, +/-, do not allow \sqrt{n} Answer in range [0.128, 0.129]
(iii)	No, distribution irrelevant	B1	1	“No” stated or implied, any valid comment
5 (i)	$H_0 : \mu = 500$ where μ denotes $H_1 : \mu < 500$ the population mean $\alpha:$ $z = \frac{435 - 500}{100 / \sqrt{4}} = -1.3$ Compare –1.282	B2 M1 A1 B1		Both hypotheses stated correctly [SR: 1 error, B1, but \bar{x} etc: B0] Standardise, use $\sqrt{4}$, can be + $z = -1.3$ (allow –1.29 from cc) <i>or</i> $\Phi(z) = 0.0968$ (.0985) Compare z & –1.282 <i>or</i> $p (< 0.5)$ & 0.1 or equivalent
	$\beta:$ $500 - 1.282 \times 100 / \sqrt{4}$ = 435.9; compare 435	M1 A1√;B1		$500 - z \times 100 / \sqrt{4}$, allow $\sqrt{\quad}$ errors, any Φ^{-1} , must be – CV correct, $\sqrt{\quad}$ on their z ; 1.282 correct and compare
	Reject H_0 Significant evidence that number of visitors has decreased	M1√ A1√	7	Correct deduction, needs $\sqrt{4}$, $\mu = 500$, like-with-like Correct conclusion interpreted in context
(ii)	CLT doesn't apply as n is small So need to know distribution	M1 B1	2	Correct reason [“ n is small” is sufficient] Refer to distribution, e.g. “if not normal, can't do it”

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6	(i)	(a) $1 - 0.8153$ $= 0.1847$ (b) $0.8153 - 0.6472$ $= \mathbf{0.168}$	M1 A1 M1 A1	2 2	Po(3) tables, "1 -" used, e.g. 0.3528 or 0.0839 Answer 0.1847 or 0.185 Subtract 2 tabular values, or formula $[e^{-3} 3^4/4!]$ Answer, a.r.t. 0.168
	(ii)	$N(150, 150)$ $1 - \Phi\left(\frac{165.5 - 150}{\sqrt{150}}\right)$ $= 1 - \Phi(1.266) = \mathbf{0.103}$	B1 B1 M1 A1 A1	5	Normal, mean 3×50 stated or implied Variance or SD = 3×50 , or same as μ Standardise 165 with λ , $\sqrt{\lambda}$ or λ , any or no cc $\sqrt{\lambda}$ and 165.5 Answer in range [0.102, 0.103]
7	(iii)	(a) The sale of one house does not affect the sale of any others (b) The average number of houses sold in a given time interval is constant	B1 B1	2	Relevant answer that shows evidence of correct understanding [but <i>not</i> just examples] Different reason, in context [Allow "constant rate" or "uniform" but not "number constant", "random", "singly", "events".]
	(i)	$\int_0^2 kx dx = \left[\frac{kx^2}{2}\right]_0^2 = 2k$ $= 1$ so $k = \frac{1}{2}$	M1 A1	2	Use $\int_0^2 kx dx = 1$, or area of triangle Correctly obtain $k = \frac{1}{2}$ AG
	(ii)		B1 B1	2	Straight line, positive gradient, through origin Correct, some evidence of truncation, no need for vertical
	(iii)	$\int_0^2 \frac{1}{2}x^2 dx = \left[\frac{1}{6}x^3\right]_0^2 = \frac{4}{3}$ $\int_0^2 \frac{1}{2}x^3 dx = \left[\frac{1}{8}x^4\right]_0^2 [= 2]$ $2 - \left(\frac{4}{3}\right) = \frac{2}{9}$	M1 A1 M1 M1 A1	5	Use $\int_0^2 kx^2 dx$; $\frac{4}{3}$ seen or implied Use $\int_0^2 kx^3 dx$; subtract their mean ² Answer $\frac{2}{9}$ or a.r.t. 0.222, c.a.o.
	(iv)		M1 A1✓	2	Translate horizontally, allow stated, or "1, 2" on axis One unit to right, 1 and 3 indicated, nothing wrong seen, no need for vertical or emphasised zero bits [If in doubt as to → or ↓, M0 in this part]
	(v)	$\frac{7}{3}$ $\frac{2}{9}$	B1✓ B1✓	2	Previous mean + 1 Previous variance [If in doubt as to → or ↓, B1B1 in this part]

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8	(i)	$H_0: p = 0.65$ OR $p \geq 0.65$ $H_1: p < 0.65$ $B(12, 0.65)$	B2 M1	Both hypotheses correctly stated, in this form [One error (but not r , x or \bar{x}): B1] $B(12, 0.65)$ stated or implied
		α : $P(\leq 6) = 0.2127$ Compare 0.10	A1 B1	Correct probability from tables, <i>not</i> $P(= 6)$ Explicit comparison with 0.10
		β : Critical region ≤ 5 ; $6 > 5$ Probability 0.0846	B1 A1	Critical region ≤ 5 or ≤ 6 or $\{\leq 4\} \cap \{\geq 11\}$ & compare 6 Correct probability
		Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65%	M1√ A1√ 7	Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. “consistent with claim” [SR: $N(7.8, 2.73)$: can get B2M1A0B1M0: 4 ex 7]
(ii)	Insufficient evidence to reject claim; test and p/q symmetric	B1√ B1 2	Same conclusion as for part (i), don't need context Valid relevant reason, e.g. “same as (i)”	
(iii)	$R \sim B(2n, 0.65)$, $P(R \leq n) > 0.15$ $B(18, 0.65)$, $p = 0.1391$ Therefore $n = 9$	M1 A1 A1 A1 4	$B(2n, 0.65)$, $P(R \leq n) > 0.15$ stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR $< n$: M1A0, $n = 4$, 0.1061 A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR $B(12, 0.35)$: M1A0, $n = 4$, 0.1061 A1A0] 3 0.3529 7 0.1836 12 0.0942 4 0.2936 8 0.1594 13 0.0832 5 0.2485 9 0.1391 14 0.0736 6 0.2127 10 0.1218 15 0.0652	