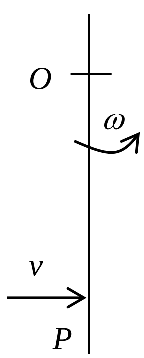
Mock Paper Mark Scheme

Advanced Subsidiary/Advanced GCE
General Certificate of Education

Subject **MECHANICS** Paper No. **Mock M5**

Question number	Scheme	Marks
1. (*a*)	Reaction **R** is perpendicular to the wire, since the wire is smooth. Hence **R.d** = 0.	B1 (1)
(*b*)	$\overrightarrow{AB} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) - (\mathbf{i} + \mathbf{j} - 3\mathbf{k}) = 2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$	M1
	Work energy: $[2\mathbf{i} + 3\mathbf{j} + (x - 4.9)\mathbf{k}].(2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) = \frac{1}{2} \times 0.5(4^2 - 2^2)$	M1 A1 A1
	$4 - 6 + 5(x - 4.9) = 3 \Rightarrow x = 5.9$	A1 (6) **(7)**
2.		
	MI of rod about O is $\frac{1}{3}m(2a)^2 + ma^2 = \frac{7}{3}ma^2$	B1
	Moment of momentum: $3mv \times x = (\frac{7}{3}ma^2 + mx^2)\omega$	M1 A1 A1 ft
	$x\omega = \frac{2}{3}v$	M1
	$\Rightarrow 3m \times \dfrac{3x\omega}{2} \times x = (\frac{7}{3}ma^2 + mx^2)\omega$	
	$\Rightarrow \frac{7}{2}x^2 = \frac{7}{3}a^2 \quad \Rightarrow \quad x = a\sqrt{\frac{2}{3}}$	M1 A1 **(7)**
3.	Auxiliary equation $m^2 + 4m + 3 = 0 \Rightarrow m = -1$ or -3	M1
	$\Rightarrow \mathbf{r} = \mathbf{A}e^{-t} + \mathbf{B}e^{-3t}$	A1
	$t = 0,\ \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow \mathbf{A} + \mathbf{B} = 2\mathbf{i}$ $t = 0,\ \dot{\mathbf{r}} = 2\mathbf{j} \Rightarrow -\mathbf{A} - 3\mathbf{B} = 2\mathbf{j}$	M1 A1
	Solving: $-2\mathbf{B} = 2\mathbf{i} + 2\mathbf{j}$ $\Rightarrow \mathbf{B} = -(\mathbf{i} + \mathbf{j})$ $\mathbf{A} = (3\mathbf{i} + \mathbf{j})$ $\mathbf{r} = (3\mathbf{i} + \mathbf{j})e^{-t} - (\mathbf{i} + \mathbf{j})e^{-3t}$	M1 A1
	$t = \ln 2 \Rightarrow e^{-t} = \frac{1}{2},\ e^{-3t} = \frac{1}{8}$	B1
	$\mathbf{r} = \frac{1}{2}(3\mathbf{i} + \mathbf{j}) - \frac{1}{8}(\mathbf{i} + \mathbf{j}) = \frac{11}{8}\mathbf{i} + \frac{3}{8}\mathbf{j}$	M1 A1 **(10)**

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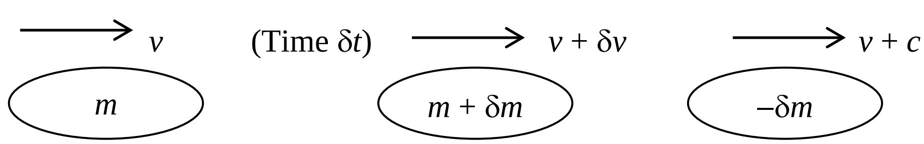
4. (*a*)	$\mathbf{F} = (2\mathbf{j} + 3\mathbf{k}) + (\mathbf{i} + 4\mathbf{k}) = \mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$	M1
	$\lvert\mathbf{F}\rvert = \sqrt{(1 + 4 + 49)} = \sqrt{54}$ N	M1 A1 (3)
(*b*)	$\mathbf{F}$ acts through point with p.v. $\mathbf{r}$	
	$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$	M1
	$\begin{pmatrix} 7y - 2z \\ z - 7x \\ 2x - y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \\ -2 \end{pmatrix}, = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$	A1 A1, A1
	so $7y - 2z = 6$, $z - 7x = -3$, $2x - y = 0$	
	$\Rightarrow y = 2x \Rightarrow$ e.g. $x = 0, y = 0, z = -3$	M1 A1
	Hence equation of line of action of $\mathbf{F}$ is	
	$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$	M1 A1 (8)
		(11)
5.		
	$(m + \delta m)(v + \delta v) + (-\delta m)(v + c) - mv = 0$	M1 A2, 1, 0
	$\Rightarrow m\,\mathrm{d}v - c\,\mathrm{d}m = 0$ reducing equation	M1
	$\int_V^{\frac{2}{3}V} \mathrm{d}v = c\int_M^m \frac{\mathrm{d}m}{m}$ separating variables	M1
	$\Big[v\Big]_V^{\frac{2}{3}V} = c\Big[\ln m\Big]_M^m$ integration	M1 A1
	$-\frac{1}{3}v = c \ln\left(\frac{m}{M}\right)$ applying limits	M1
	$\Rightarrow m = Me^{-\frac{V}{3c}}$ eliminating ln and m =	M1
	$\therefore$ Fuel used $= M - m = M(1 - e^{-\frac{V}{3c}})$	M1 A1 **(11)**

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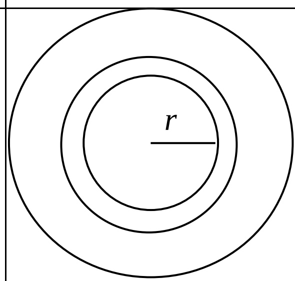
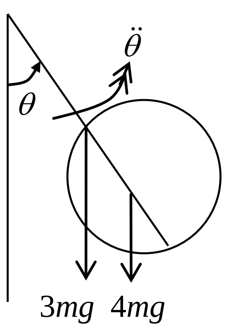
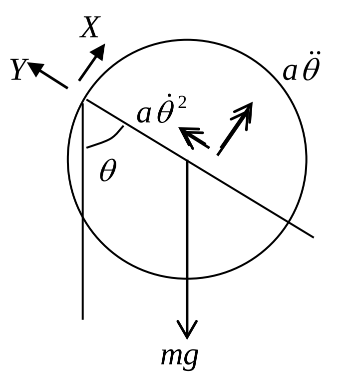
6. (*a*)	 MI of element = $2\pi\rho r\delta r \times r^2$	M1
	$m = \pi\rho a^2$	
	$\Rightarrow \mathrm{I} = \dfrac{2m}{a^2}\displaystyle\int_0^a r^3\,\mathrm{d}r$	M1
	$= \dfrac{2m}{a^2}\left[\dfrac{r^4}{4}\right]_0^a = \tfrac{1}{2}ma^2$	A1 (3)
(*b*)	$\mathrm{I} = \mathrm{I}_{\text{rod}} + \mathrm{I}_{\text{disc}} = \tfrac{4}{3} \times 3m \times a^2, + \tfrac{1}{2} \times 4m\left(\dfrac{a}{2}\right)^2 + 4m\left(\dfrac{3a}{2}\right)^2$	B1, M1
	$= 4ma^2 + \dfrac{ma^2}{2} + 9ma^2$	
	$= \tfrac{27}{2}ma^2$	A1 (3)
(*c*)	 $\tfrac{27}{2}ma^2\,\ddot{\theta} = -3mg \times a\sin\theta - 4mg \times \dfrac{3a}{2}\sin\theta$	M1 A2, 1, 0
	$= -9mga\sin\theta$	
	$\ddot{\theta} = -\dfrac{2g}{3a}\sin\theta$	
	Small oscillations $\Rightarrow \sin\theta \approx \theta$	M1
	$\Rightarrow \ddot{\theta} = -\dfrac{2g}{3a}\theta$ which is SHM	A1
	$T = 2\pi\sqrt{\dfrac{3a}{2g}}$	A1 (6)
		(12)

Question number	Scheme	Marks
7. (*a*)	MI of sphere about $L = \frac{2}{5}ma^2 + ma^2 = \frac{7}{5}ma^2$	B1
	Energy: $\frac{1}{2}\times\frac{7}{5}ma^2 \times \frac{18g}{7a} - \frac{1}{2}\times\frac{7}{5}ma^2 \times \omega^2 = mga(1 - \cos\theta)$	M1 A2, 1, 0
	$\Rightarrow \frac{7}{10}a\omega^2 = \frac{8}{10}g + g\cos\theta$	
	$\omega^2 = \frac{g}{7a}(8 + 10\cos\theta) = \frac{2g}{7a}(4 + 5\cos\theta)$	A1 (5)
(*b*)	$\frac{7}{5}ma^2\ddot{\theta} = -mga\sin\theta \Rightarrow \ddot{\theta} = -\frac{5g}{7a}\sin\theta$	M1 A1 (2)
	[or $2\omega\dot{\omega} = -\frac{10g}{7a}\sin\theta\times\omega \Rightarrow \dot{\omega} = -\frac{5g}{7a}\sin\theta$]	
(*c*)	$\dot{\theta} = 0$ when $\cos\theta = -\frac{4}{5}$	B1
		
	$Y - mg\cos\theta = ma\dot{\theta}^2$	M1
	$\dot{\theta} = 0, \cos\theta = -\frac{4}{5} \Rightarrow Y = -\frac{4mg}{5}$	M1 A1
	$X - mg\sin\theta = ma\ddot{\theta}$	M1
	$\dot{\theta} = 0$ and $\cos\theta = -\frac{4}{5} \Rightarrow \sin\theta = \frac{3}{5}$	M1
	$\Rightarrow \ddot{\theta} = -\frac{3g}{7a}$	
	$\Rightarrow X = \frac{3mg}{5} - \frac{3mg}{7} = \frac{6mg}{35}$	A1
	Magnitude of force $= \sqrt{(X^2 + Y^2)}$	M1
	$= mg\left[\left(\frac{6}{35}\right)^2 + \left(\frac{4}{5}\right)^2\right]^{\frac{1}{2}}$	
	$\approx 0.818\,mg$	A1 (10)
		(17)