D2 2003 (adapted for new spec)

1. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays I	*B* plays II	*B* plays III
A plays I	–3	2	5
A plays II	4	–1	–4

(a) Write down the pay off matrix for player *B*.

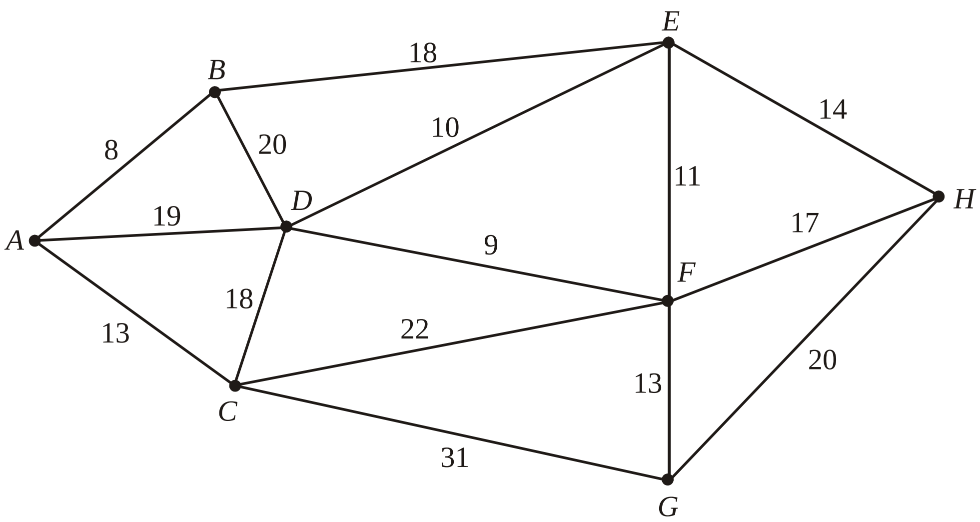
(2)

(b) Formulate the game as a linear programming problem for player *B*, writing the constraints as equalities and stating your variables clearly.

(4)
(Total 6 marks)

2. (a) Explain the difference between the classical and practical travelling salesman problems.

(2)



The network in the diagram above shows the distances, in kilometres, between eight McBurger restaurants. An inspector from head office wishes to visit each restaurant. His route should start and finish at *A*, visit each restaurant at least once and cover a minimum distance.

(b) Obtain a minimum spanning tree for the network using Kruskal's algorithm. You should draw your tree and state the order in which the arcs were added.

(3)

(c) Use your answer to part (b) to determine an initial upper bound for the length of the route.

(2)

(d) Starting from your initial upper bound and using an appropriate method, find an upper bound which is less than 135 km. State your tour.

(3)
(Total 10 marks)

3. Talkalot College holds an induction meeting for new students. The meeting consists of four talks: I (Welcome), II (Options and Facilities), III (Study Tips) and IV (Planning for Success). The four department heads, Clive, Julie, Nicky and Steve, deliver one of these talks each. The talks are delivered consecutively and there are no breaks between talks. The meeting starts at 10 a.m. and ends when all four talks have been delivered. The time, in minutes, each department head takes to deliver each talk is given in the table below.

	Talk I	Talk II	Talk III	Talk IV
Clive	12	34	28	16
Julie	13	32	36	12
Nicky	15	32	32	14
Steve	11	33	36	10

(a) Use the Hungarian algorithm to find the earliest time that the meeting could end. You must make your method clear and show

(i) the state of the table after each stage in the algorithm,

(ii) the final allocation.

(10)

(b) Modify the table so it could be used to find the latest time that the meeting could end.

(3)

(Total 13 marks)

4. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays I	*B* plays II	*B* plays III
A plays I	2	–1	3
A plays II	1	3	0
A plays III	0	1	–3

(a) Identify the play safe strategies for each player.

(4)

(b) Verify that there is no stable solution to this game.

(1)

(c) Explain why the pay-off matrix above may be reduced to

	B plays I	*B* plays II	*B* plays III
A plays I	2	–1	3
A plays II	1	3	0

(2)

(d) Find the best strategy for player *A*, and the value of the game.

(7)

(Total 14 marks)

5. The manager of a car hire firm has to arrange to move cars from three garages *A*, *B* and *C* to three airports *D*, *E* and *F* so that customers can collect them. The table below shows the transportation cost of moving one car from each garage to each airport. It also shows the number of cars available in each garage and the number of cars required at each airport. The total number of cars available is equal to the total number required.

	Airport *D*	Airport *E*	Airport *F*	Cars available
Garage *A*	£20	£40	£10	6
Garage *B*	£20	£30	£40	5
Garage *C*	£10	£20	£30	8
Cars required	6	9	4	

(a) Use the North-West corner rule to obtain a possible pattern of distribution and find its cost.

(3)

(b) Calculate shadow costs for this pattern and hence obtain improvement indices for each route.

(4)

(c) Use the stepping-stone method to obtain an optimal solution and state its cost.

(7)
(Total 14 marks)

6. Kris produces custom made racing cycles. She can produce up to four cycles each month, but if she wishes to produce more than three in any one month she has to hire additional help at a cost of £350 for that month. In any month when cycles are produced, the overhead costs are £200. A maximum of 3 cycles can be held in stock in any one month, at a cost of £40 per cycle per month. Cycles must be delivered at the end of the month. The order book for cycles is

Month	August	September	October	November
Number of cycles required	3	3	5	2

Disregarding the cost of parts and Kris' time,

(a) determine the total cost of storing 2 cycles and producing 4 cycles in a given month, making your calculations clear.

(2)

There is no stock at the beginning of August and Kris plans to have no stock after the November delivery.

(b) Use dynamic programming to determine the production schedule which minimises the costs, showing your working in the table below.

Stage	Demand	State	Action	Destination	Value
1 (Nov)	2	0 (in stock)	(make) 2	0	200
		1 (in stock)	(make) 1	0	240
		2 (in stock)	(make) 0	0	80
2 (Oct)	5	1	4	0	590 + 200 = 790
		2	3	0	
			4	1	

(13)

The fixed cost of parts is £600 per cycle and of Kris' time is £500 per month. She sells the cycles for £2000 each.

(c) Determine her total profit for the four month period.

(3)
(Total 18 marks)

7.

Figure 1

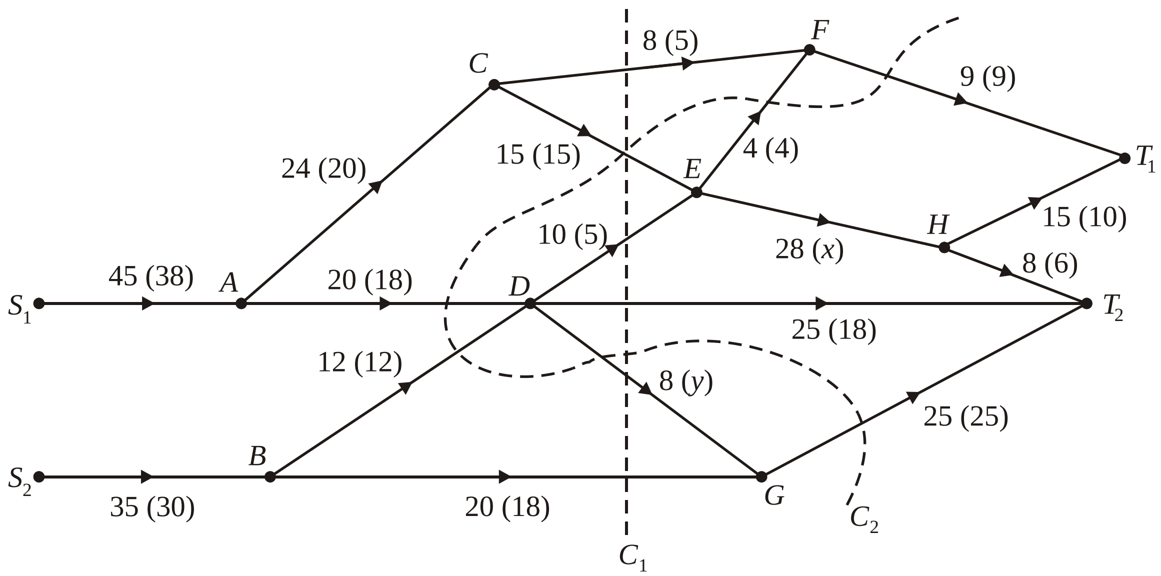
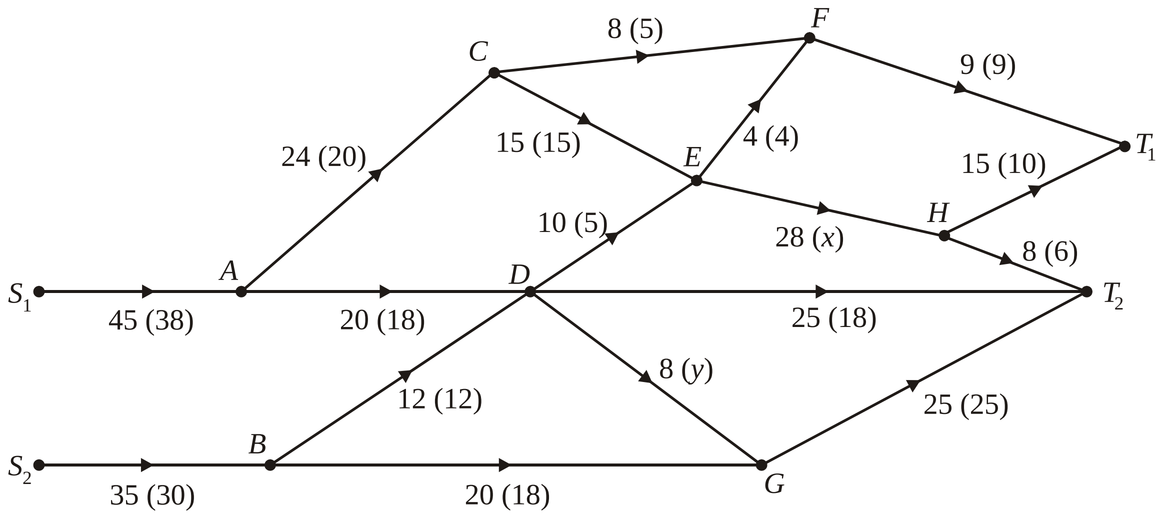


Figure 1 shows a capacitated, directed network. The unbracketed number on each arc indicates the capacity of that arc, and the numbers in brackets show a feasible flow of value 68 through the network.

(a) Add a supersource and a supersink, and arcs of appropriate capacity, to Diagram 1 below.

Diagram 1



(2)

(b) Find the values of x and y, explaining your method briefly.

(2)

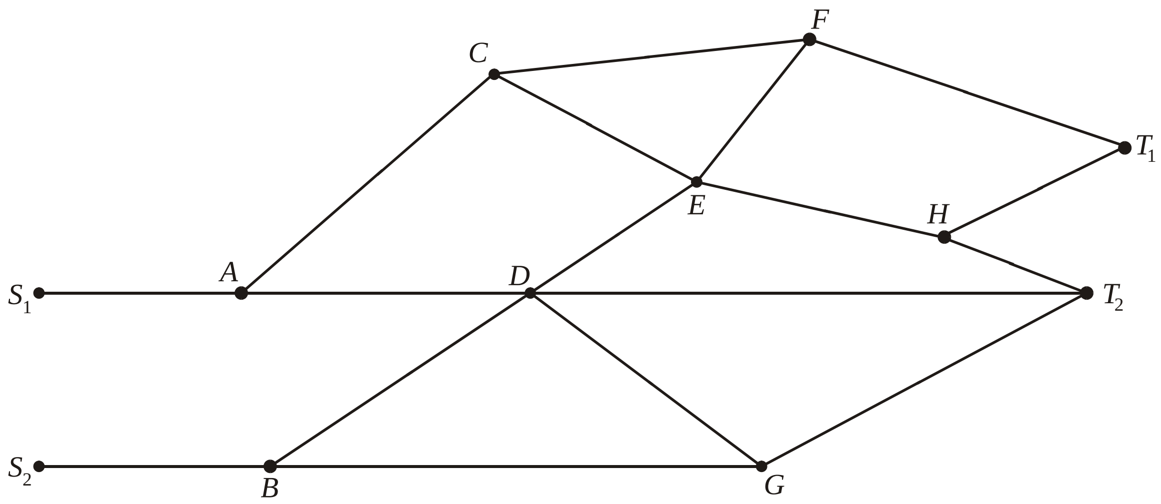
(c) Find the value of cuts C_1 and C_2.

(3)

Starting with the given feasible flow of 68,

(d) use the labelling procedure on Diagram 2 to find a maximal flow through this network. List each flow-augmenting route you use, together with its flow.

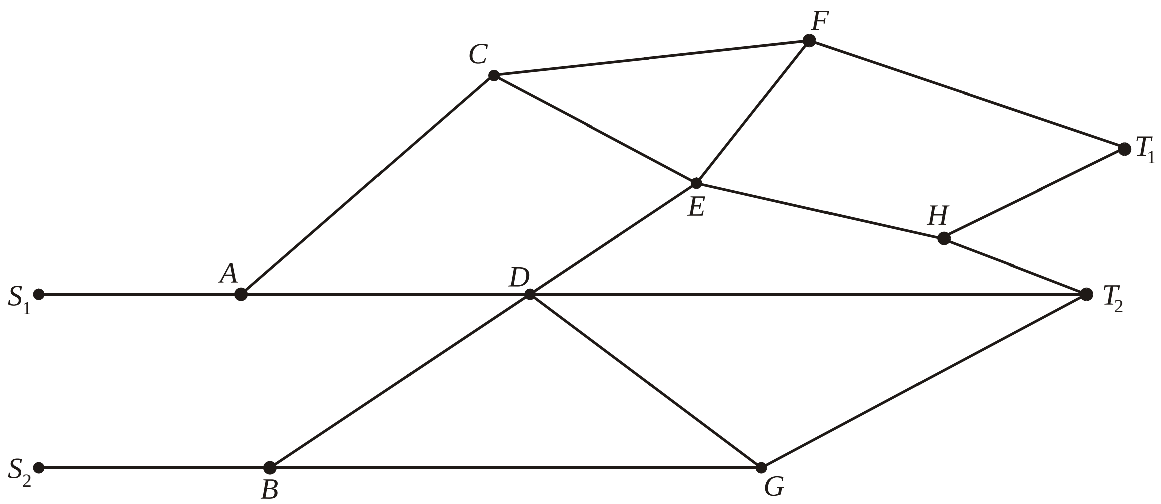
Diagram 2



(6)

(e) Show your maximal flow on Diagram 3 and state its value.

Diagram 3



(3)

(f) Prove that your flow is maximal.

(2)
(Total 18 marks)

8. The tableau below is the initial tableau for a maximising linear programming problem.

Basic variable	x	y	z	r	s	Value
r	2	3	4	1	0	8
s	3	3	1	0	1	10
P	–8	–9	–5	0	0	0

(a) For this problem $x \geq 0$, $y \geq 0$, $z \geq 0$. Write down the other two inequalities and the objective function.

(3)

(b) Solve this linear programming problem.

You may not need to use all of these tableaux.

b.v.	x	y	z	r	s	Value
P						

b.v.	x	y	z	r	s	Value
P						

b.v.	x	y	z	r	s	Value
P						

b.v.	x	y	z	r	s	Value
P						

(8)

(c) State the final value of P, the objective function, and of each of the variables.

(3)

(Total 14 marks)