

# **Mark Scheme 4736 June 2007**

SOLUTIONS

4736

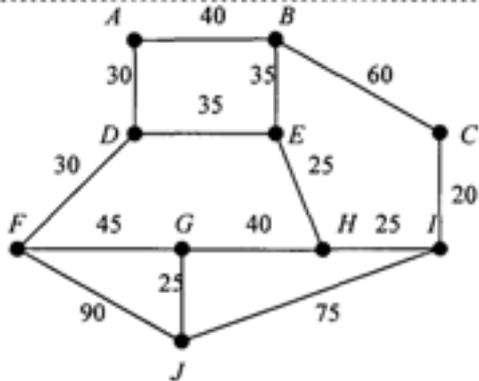
D1

June 2007

FINAL

1	(i)	Example: $N-P-Q-T-S-R-N$ or: $P-Q-S-P$	B1	1	Any valid cycle (closed and does not repeat vertices, need not be a Hamiltonian cycle)																									
	(ii)	It passes through $Y$ twice	B1	1	Or, it includes a cycle (accept 'loop')																									
	(iii)	5	B1	1																										
	(iv)	$A$ : neither $B$ : semi-Eulerian	B1	2	If graphs are not specified, assume $A$ is first																									
	(v)	$A$ : 2 $B$ : 1	B1	2	If graphs are not specified, assume $A$ is first																									
	(vi)	There are 4 odd nodes ( $N, P, S$ and $Z$ ) To connect these we must add 2 arcs	M1	2	$A$ : 1, $B$ : 0 $\Rightarrow$ B1 only Seen or implied																									
		A1	9	For 2																										
2	(i)	$d+f+g=120$	B1	1	For this equality. Condone an inequality																									
	(ii)	"(Area of) grass is not more than 4 times (area of) decking"	B1	1	Identifying the constraint in words (not just 'grass is less than or equal to 4 times decking' though)																									
	(iii)	$d \leq f$	B1	1	Do not accept $d < f$																									
	(iv)	$g \geq 40$ $\min d = 10$ $\min f = 20$	B1	3	Do not accept $g > 40$ $d \geq 10$ $f \geq 20$																									
	(v)	$5g + 10d + 20f$ or $g + 2d + 4f$	B1	1	Or any positive multiple of this																									
	(vi)	Minimise $g + 2d + 4f$ Subject to $d + f + g = 120$ $g - 4d + s = 0$ $d - f + t = 0$ $g \geq 40$ , and $d \geq 10, f \geq 20, s \geq 0, t \geq 0$	M1	3	For a reasonable attempt at setting up the minimisation problem using their expressions																									
		B1		For dealing with this slack variable correctly (variables on LHS and constant on RHS)																										
		A1	10	For a completely correct formulation (accept $d$ and $f \geq 0$ , or their min values for $d, f$ )																										
3	(i)	<table border="0"> <tr> <td></td> <td>8 6 9 7 5</td> <td>Comps</td> <td>Swaps</td> <td></td> </tr> <tr> <td>After 1st pass:</td> <td>6 8 9 7 5</td> <td>1</td> <td>1</td> <td>M1</td> </tr> <tr> <td>After 2nd pass:</td> <td>6 8 9 7 5</td> <td>1</td> <td>0</td> <td>M1</td> </tr> <tr> <td>After 3rd pass:</td> <td>6 7 8 9 5</td> <td>3</td> <td>2</td> <td>M1</td> </tr> <tr> <td>After 4th pass:</td> <td>5 6 7 8 9</td> <td>4</td> <td>4</td> <td>A1</td> </tr> </table> <p>Comparisons must be 1, 2, 3 or 4 with total <math>\leq 10</math> Swaps must be 0, 1, 2, 3 or 4 and no more than corresponding number of comparisons</p>		8 6 9 7 5	Comps	Swaps		After 1st pass:	6 8 9 7 5	1	1	M1	After 2nd pass:	6 8 9 7 5	1	0	M1	After 3rd pass:	6 7 8 9 5	3	2	M1	After 4th pass:	5 6 7 8 9	4	4	A1	B1	6	Bubble sort or decreasing order loses first 4 marks 1st pass correct 2nd pass correct, follow through from 1st pass 3rd pass correct, follow through from 2nd pass 4th pass correct Counting comparisons for at least three passes Counting swaps for at least three passes
		8 6 9 7 5	Comps	Swaps																										
After 1st pass:	6 8 9 7 5	1	1	M1																										
After 2nd pass:	6 8 9 7 5	1	0	M1																										
After 3rd pass:	6 7 8 9 5	3	2	M1																										
After 4th pass:	5 6 7 8 9	4	4	A1																										
(ii)	<p>Step 1 A = 8 6 9 7 5</p> <p>Step 2 A = 6 9 7 5 X = 8</p> <p>Step 3 A = 9 7 5 B = 6</p> <p>Step 4 A = 7 5 C = 9</p> <p>Step 4 A = 5 B = 6 7</p> <p>Step 4 A is empty B = 6 7 5</p> <p>Step 6 N = 3</p> <p>Step 7 A = 6 7 5 8 9</p> <p>Step 8 Display 6 7 5 8 9</p>	M1	5	For identifying that $6 \rightarrow B$ or the sublist {6}																										
		M1		For identifying that $9 \rightarrow C$ or the sublist {9}																										
		M1		For identifying that $7 \rightarrow B$																										
		M1		For identifying that $5 \rightarrow B$																										
		A1	11	For the final A list or the display correct																										

<b>4</b>	<b>(i)</b>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>s</i></th> <th><i>t</i></th> <th><i>u</i></th> <th></th> </tr> <tr> <td>1</td> <td>-3</td> <td>5</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>5</td> <td>1</td> <td>0</td> <td>0</td> <td>12</td> </tr> <tr> <td>0</td> <td>1</td> <td>-5</td> <td>0</td> <td>1</td> <td>0</td> <td>10</td> </tr> <tr> <td>0</td> <td>3</td> <td>10</td> <td>0</td> <td>0</td> <td>1</td> <td>45</td> </tr> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>		1	-3	5	0	0	0	0	0	1	5	1	0	0	12	0	1	-5	0	1	0	10	0	3	10	0	0	1	45	<p>B1 For correct use of three slack variable columns</p> <p>B1 For <math>\pm (-3 \ 5)</math> in objective row</p> <p>B1 For 1 5 12, 1 -5 10 and 3 10 45 in constraint rows</p> <p style="text-align: right;"><b>3</b></p>
	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>																																
	1	-3	5	0	0	0	0																															
	0	1	5	1	0	0	12																															
0	1	-5	0	1	0	10																																
0	3	10	0	0	1	45																																
<b>(ii)</b>	<p>Pivot on second 1 in <i>x</i> column  <i>x</i> column has a negative entry in objective row  <math>12 \div 1 = 12</math>, <math>10 \div 1 = 10</math>, <math>45 \div 3 = 15</math>                      Least non-negative ratio is 10 so pivot on the second 1</p>	<p>B1 For correct pivot choice (cao)</p> <p>B1 For 'negative in top row for <i>x</i>', or equivalent, and a correct explanation of choice of row 'least ratio <math>10 \div 1</math>' (ft their pivot column)</p> <p style="text-align: right;"><b>2</b></p>																																				
<b>(iii)</b>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>z</i></th> <th><i>s</i></th> <th><i>t</i></th> <th></th> </tr> <tr> <td>1</td> <td>0</td> <td>-10</td> <td>0</td> <td>3</td> <td>0</td> <td>30</td> </tr> <tr> <td>0</td> <td>0</td> <td>10</td> <td>1</td> <td>-1</td> <td>0</td> <td>2</td> </tr> <tr> <td>0</td> <td>1</td> <td>-5</td> <td>0</td> <td>1</td> <td>0</td> <td>10</td> </tr> <tr> <td>0</td> <td>0</td> <td>25</td> <td>0</td> <td>-3</td> <td>1</td> <td>15</td> </tr> </table> <p><math>x = 10, y = 0</math>  <math>P = 30</math></p>	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i>	<i>t</i>		1	0	-10	0	3	0	30	0	0	10	1	-1	0	2	0	1	-5	0	1	0	10	0	0	25	0	-3	1	15	<p>ft their tableau if possible for method marks</p> <p>M1 For correct method evident for objective row</p> <p>M1 For a correct method evident for pivot row</p> <p>M1 For a correct method evident for other rows</p> <p>A1 For correct tableau CAO</p> <p>B1 For correct values from their tableau</p> <p>B1 <b>6</b> For correct value from their tableau</p>	
<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i>	<i>t</i>																																	
1	0	-10	0	3	0	30																																
0	0	10	1	-1	0	2																																
0	1	-5	0	1	0	10																																
0	0	25	0	-3	1	15																																
<b>(iv)</b>	<p><math>11 + 5(0.2) = 12</math> or <math>s = 0</math>  <math>11 - 5(0.2) = 10</math> or <math>t = 0</math>  <math>3(11) + 10(0.2) = 35</math> or <math>u = 10</math>                      so all the constraints are satisfied</p> <p><math>P = 3(11) - 5(0.2) = 32</math>                      which is bigger than 30 from (iii)</p>	<p>B1 For showing (not just stating) that constraints are satisfied</p> <p>B1 <b>2</b> For calculating 32, or equivalent (eg 3<i>x</i> has increased by 3 but -5<i>y</i> has only decreased by 1)</p> <p style="text-align: right;"><b>13</b></p>																																				

<p>5</p>	<p>(i)</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p><i>A</i></p> <table border="1" style="border-collapse: collapse; width: 40px; height: 40px;"> <tr><td style="width: 20px; height: 20px;"></td><td style="width: 20px; height: 20px;"></td></tr> <tr><td colspan="2" style="text-align: center;">130</td></tr> </table> </div> <div style="text-align: center;"> <p><i>B</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">9</td><td style="width: 30px; height: 20px;">125</td></tr> <tr><td colspan="2" style="text-align: center;">125</td></tr> </table> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <p><i>D</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">8</td><td style="width: 30px; height: 20px;">100</td></tr> <tr><td colspan="2" style="text-align: center;">100</td></tr> </table> </div> <div style="text-align: center;"> <p><i>E</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">6</td><td style="width: 30px; height: 20px;">90</td></tr> <tr><td colspan="2" style="text-align: center;">90</td></tr> </table> </div> <div style="text-align: center;"> <p><i>C</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">7</td><td style="width: 30px; height: 20px;">95</td></tr> <tr><td colspan="2" style="text-align: center;">95</td></tr> </table> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <p><i>F</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">4</td><td style="width: 30px; height: 20px;">70</td></tr> <tr><td colspan="2" style="text-align: center;">90 70</td></tr> </table> </div> <div style="text-align: center;"> <p><i>G</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">2</td><td style="width: 30px; height: 20px;">25</td></tr> <tr><td colspan="2" style="text-align: center;">25</td></tr> </table> </div> <div style="text-align: center;"> <p><i>H</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">3</td><td style="width: 30px; height: 20px;">65</td></tr> <tr><td colspan="2" style="text-align: center;">65</td></tr> </table> </div> <div style="text-align: center;"> <p><i>I</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">5</td><td style="width: 30px; height: 20px;">75</td></tr> <tr><td colspan="2" style="text-align: center;">75</td></tr> </table> </div> </div> <div style="text-align: center; margin-top: 20px;"> <p><i>J</i></p> <table border="1" style="border-collapse: collapse; width: 60px; height: 40px;"> <tr><td style="width: 30px; height: 20px;">1</td><td style="width: 30px; height: 20px;">0</td></tr> <tr><td colspan="2" style="text-align: center;"> </td></tr> </table> </div> <p style="margin-top: 20px;">Shortest path from <i>J</i> to <i>B</i>: <i>J G H E B</i> Length of path: 125 metres</p>			130		9	125	125		8	100	100		6	90	90		7	95	95		4	70	90 70		2	25	25		3	65	65		5	75	75		1	0			<p>ANSWERED ON INSERT</p> <p>M1 For correct initial temporary labels at <i>F, G, I</i></p> <p>M1 For correctly updating <i>F</i> and label at <i>H</i></p> <p>A1 For all temporary labels correct (including <i>A</i>) (allow extra 100 at <i>C</i>, 105 at <i>D</i>, 75 at <i>H</i> only)</p> <p>B1 For order of becoming permanent correct</p> <p>B1 For all permanent labels correct (<i>A</i> need not have a permanent label)</p> <p>B1 For correct route (condone omission of <i>J</i> or <i>B</i>)</p> <p>B1 7 For 125</p>
130																																										
9	125																																									
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8	100																																									
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6	90																																									
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3	65																																									
65																																										
5	75																																									
75																																										
1	0																																									
<p>(ii)</p>	<p>Odd nodes: <i>B C E J</i></p> <p><math>BC = 60</math>   <math>BE = 35</math>   <math>BJ = 125</math>  <math>EJ = 90</math>   <math>CJ = 95</math>   <math>CE = 70</math>                150       130       195</p> <p>Repeat <i>BE</i> and <i>CJ</i> (or <i>BE, JI, IC</i>)</p> <p>130 + 765</p> <p>Shortest route: 895 metres</p>	<p>B1 For identifying or using <i>B C E J</i> or implied</p> <p>M1 For any three of these weights correct, or implied or ft from their (i)</p> <p>A1 For identifying the pairing <i>BE, CJ</i> to repeat or 130 (not ft)</p> <p>M1 For 765 + their 130 (a valid pairs total)</p> <p>A1 5 For 895 (cao)</p>																																								
<p>(iii)</p>	 <p style="margin-top: 20px;">Travelling salesperson problem</p>	<p>B1 For graph structure correct</p> <p>M1 For a reasonable attempt at arc weights (at least 9 correct, including the three given)</p> <p>A1 For all arc weights correct</p> <p>B1 4 For identifying TSP by name</p>																																								

<b>6</b>	(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">1</td> <td style="text-align: center;">5</td> <td style="text-align: center;">2</td> <td style="text-align: center;">4</td> <td style="text-align: center;">3</td> <td style="text-align: center;">6</td> </tr> <tr> <td style="text-align: center;">A</td> <td style="text-align: center;">-</td> <td style="text-align: center;">6</td> <td style="text-align: center;">3</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> </tr> <tr> <td style="text-align: center;">B</td> <td style="text-align: center;">6</td> <td style="text-align: center;">-</td> <td style="text-align: center;">5</td> <td style="text-align: center;">6</td> <td style="text-align: center;">-</td> <td style="text-align: center;">14</td> </tr> <tr> <td style="text-align: center;">C</td> <td style="text-align: center;">3</td> <td style="text-align: center;">5</td> <td style="text-align: center;">-</td> <td style="text-align: center;">8</td> <td style="text-align: center;">4</td> <td style="text-align: center;">10</td> </tr> <tr> <td style="text-align: center;">D</td> <td style="text-align: center;">-</td> <td style="text-align: center;">6</td> <td style="text-align: center;">8</td> <td style="text-align: center;">-</td> <td style="text-align: center;">3</td> <td style="text-align: center;">8</td> </tr> <tr> <td style="text-align: center;">E</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">4</td> <td style="text-align: center;">3</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> </tr> <tr> <td style="text-align: center;">F</td> <td style="text-align: center;">-</td> <td style="text-align: center;">14</td> <td style="text-align: center;">10</td> <td style="text-align: center;">8</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> </tr> </table>		1	5	2	4	3	6	A	-	6	3	-	-	-	B	6	-	5	6	-	14	C	3	5	-	8	4	10	D	-	6	8	-	3	8	E	-	-	4	3	-	-	F	-	14	10	8	-	-		ANSWERED ON INSERT
			1	5	2	4	3	6																																													
		A	-	6	3	-	-	-																																													
		B	6	-	5	6	-	14																																													
		C	3	5	-	8	4	10																																													
		D	-	6	8	-	3	8																																													
		E	-	-	4	3	-	-																																													
		F	-	14	10	8	-	-																																													
			Order: <i>A C E D B F</i>	M1	For choosing row C in column A																																																
			Minimum spanning tree:	M1 dep	For choosing more than one entry from column C																																																
		A1	For correct entries chosen																																																		
	Total weight: 23 miles	B1	For correct order, listed or marked on arrows or table, or arcs listed <i>AC CE ED CB DF</i>																																																		
		B1	For tree (correct or follow through from table, provided solution forms a spanning tree)																																																		
		B1	For 23 (or follow through from table or diagram, provided solution forms a spanning tree)																																																		
		<b>6</b>																																																			
	(ii)	M1	For their 18 seen or implied																																																		
	MST for reduced network = 18	M1	For 11 seen or implied																																																		
	Two shortest arcs from B = 5 + 6 = 11	A1	For 29 (cao)																																																		
	Lower bound = 29 miles	<b>3</b>																																																			
	(iii)	M1	For <i>F-D-E-C-A-B</i>																																																		
	<i>F-D-E-C-A-B</i>	A1	For correct tour																																																		
	8 + 3 + 4 + 3 + 6 + 14	M1	For a substantially correct attempt at sum																																																		
	= 38 miles	A1	For 38 (cao)																																																		
		<b>13</b>																																																			

**4736**

**Mark Scheme**

**June 2007**