General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MFP2

Unit Further Pure 2

Thursday 31 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) Sketch the curve $y = \cosh x$. *(1 mark)*

(b) Solve the equation

$$6\cosh^2 x - 7\cosh x - 5 = 0$$

giving your answers in logarithmic form. *(6 marks)*

2 (a) Draw on the Argand diagram below:

(i) the locus of points for which

$$|z - 2 - 3\mathrm{i}| = 2$$ *(3 marks)*

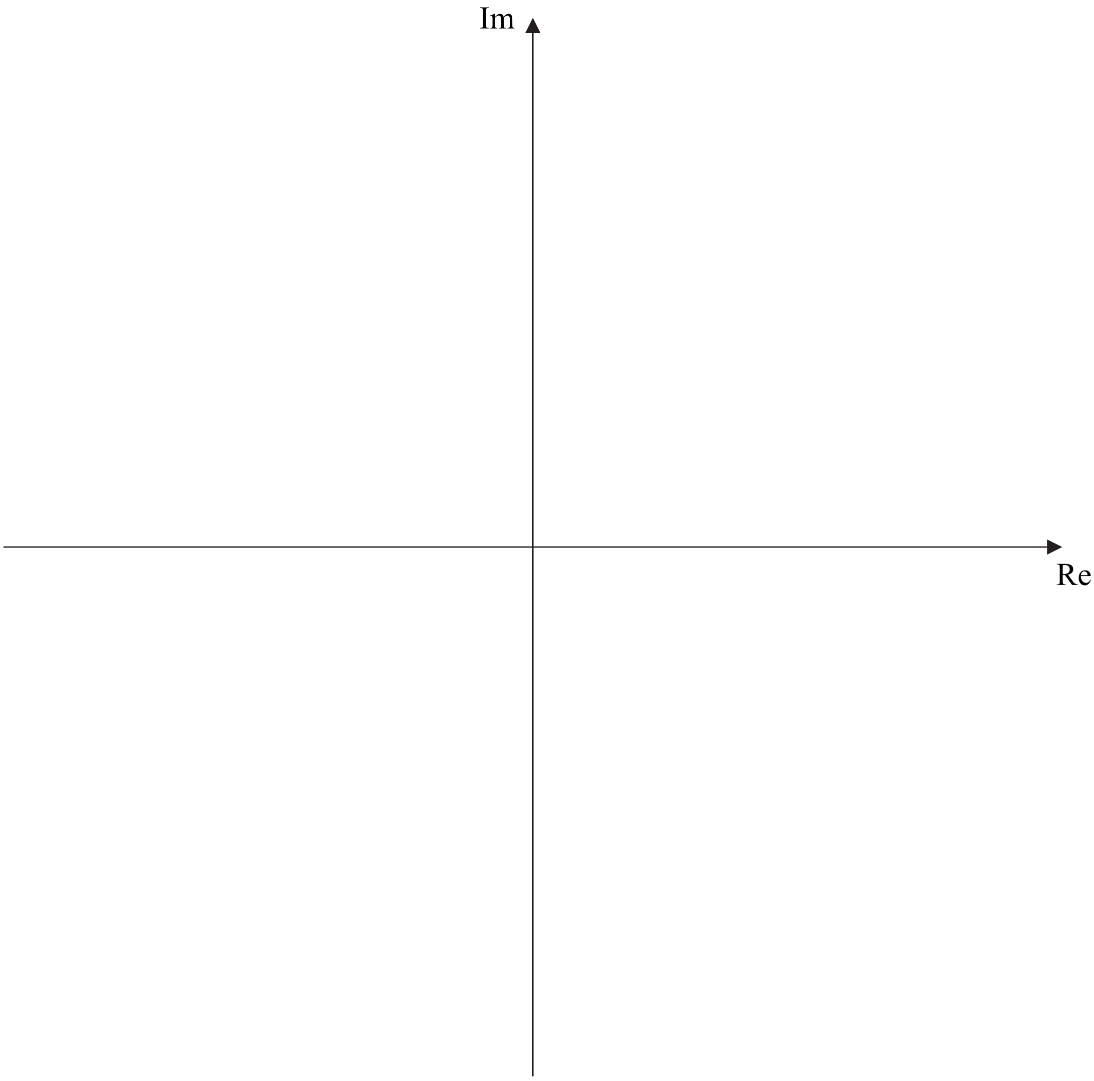
(ii) the locus of points for which

$$|z + 2 - \mathrm{i}| = |z - 2|$$ *(3 marks)*

(b) Indicate on your diagram the points satisfying both

$$|z - 2 - 3\mathrm{i}| = 2$$

and $$|z + 2 - \mathrm{i}| \leqslant |z - 2|$$ *(1 mark)*



3 (a) Show that

$$\frac{2^{r+1}}{r+2} - \frac{2^r}{r+1} = \frac{r2^r}{(r+1)(r+2)}$$ *(3 marks)*

(b) Hence find

$$\sum_{r=1}^{30} \frac{r2^r}{(r+1)(r+2)}$$

giving your answer in the form $2^n - 1$, where n is an integer. *(3 marks)*

---

4 The cubic equation

$$z^3 + pz + q = 0$$

has roots α, β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$. *(1 mark)*

(ii) Express $\alpha\beta\gamma$ in terms of q. *(1 mark)*

(b) Show that

$$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$ *(3 marks)*

(c) Given that $\alpha = 4 + 7\mathrm{i}$ and that p and q are real, find the values of:

(i) β and γ; *(2 marks)*

(ii) p and q. *(3 marks)*

(d) Find a cubic equation with integer coefficients which has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. *(3 marks)*

---

5 The function f, where $\mathrm{f}(x) = \sec x$, has domain $0 \leqslant x < \frac{\pi}{2}$ and has inverse function f^{-1}, where $\mathrm{f}^{-1}(x) = \sec^{-1} x$.

(a) Show that

$$\sec^{-1} x = \cos^{-1} \frac{1}{x}$$ *(2 marks)*

(b) Hence show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sec^{-1} x) = \frac{1}{\sqrt{x^4 - x^2}}$$ *(4 marks)*

6 (a) Show that

$$\frac{1}{4}(\cosh 4x + 2\cosh 2x + 1) = \cosh^2 x \cosh 2x$$ *(3 marks)*

(b) Show that, if $y = \cosh^2 x$, then

$$1 + \left(\frac{dy}{dx}\right)^2 = \cosh^2 2x$$ *(3 marks)*

(c) The arc of the curve $y = \cosh^2 x$ between the points where $x = 0$ and $x = \ln 2$ is rotated through 2π radians about the x-axis. Show that the area S of the curved surface formed is given by

$$S = \frac{\pi}{256}(a \ln 2 + b)$$

where a and b are integers. *(7 marks)*

---

7 (a) Prove by induction that, for all integers $n \geqslant 1$,

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots + \frac{2n+1}{n^2(n+1)^2} = 1 - \frac{1}{(n+1)^2}$$ *(7 marks)*

(b) Find the smallest integer n for which the sum of the series differs from 1 by less than 10^{-5}. *(2 marks)*

8 (a) Use De Moivre's Theorem to show that, if $z = \cos\theta + \mathrm{i}\sin\theta$, then

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

(3 marks)

(b) (i) Expand $\left(z^2 + \frac{1}{z^2}\right)^4$. *(1 mark)*

(ii) Show that

$$\cos^4 2\theta = A\cos 8\theta + B\cos 4\theta + C$$

where A, B and C are rational numbers. *(4 marks)*

(c) Hence solve the equation

$$8\cos^4 2\theta = \cos 8\theta + 5$$

for $0 \leqslant \theta \leqslant \pi$, giving each solution in the form $k\pi$. *(3 marks)*

(d) Show that

$$\int_0^{\frac{\pi}{2}} \cos^4 2\theta \, \mathrm{d}\theta = \frac{3\pi}{16}$$

(3 marks)