



Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS

4727 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The plane p has equation $\mathbf{r} \cdot (\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 4$ and the line l_1 has equation $\mathbf{r} = 2\mathbf{j} - \mathbf{k} + t(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The line l_2 is parallel to p and perpendicular to l_1 , and passes through the point with position vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Find the equation of l_2 , giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [4]

- 2 (i) Solve the equation $z^4 = 2(1 + i\sqrt{3})$, giving the roots exactly in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 \leq \theta < 2\pi$. [5]

- (ii) Sketch an Argand diagram to show the lines from the origin to the point representing $2(1 + i\sqrt{3})$ and from the origin to the points which represent the roots of the equation in part (i). [3]

- 3 Find the solution of the differential equation

$$\frac{dy}{dx} + y \cot x = 2x$$

for which $y = 2$ when $x = \frac{1}{6}\pi$. Give your answer in the form $y = f(x)$. [9]

- 4 The elements a, b, c, d are combined according to the operation table below, to form a group G of order 4.

	a	b	c	d
a	b	a	d	c
b	a	b	c	d
c	d	c	a	b
d	c	d	b	a

Group G is isomorphic **either** to the multiplicative group $H = \{e, r, r^2, r^3\}$ **or** to the multiplicative group $K = \{e, p, q, pq\}$. It is given that $r^4 = e$ in group H and that $p^2 = q^2 = e$ in group K , where e denotes the identity in each group.

- (i) Write down the operation tables for H and K . [4]

- (ii) State the identity element of G . [1]

- (iii) Demonstrate the isomorphism between G and either H or K by listing how the elements of G correspond to the elements of the other group. If the correspondence can be shown in more than one way, list the alternative correspondence(s). [4]

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- 5 (i) By expressing $\sin \theta$ and $\cos \theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, prove that

$$\sin^3 \theta \cos^2 \theta \equiv -\frac{1}{16}(\sin 5\theta - \sin 3\theta - 2 \sin \theta). \quad [6]$$

- (ii) Hence show that all the roots of the equation

$$\sin 5\theta = \sin 3\theta + 2 \sin \theta$$

are of the form $\theta = \frac{n\pi}{k}$, where n is any integer and k is to be determined. [3]

- 6 The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} = 12e^{2x}.$$

- (i) Find the general solution of the differential equation. [6]

- (ii) It is given that the curve which represents a particular solution of the differential equation has gradient 6 when $x = 0$, and approximates to $y = e^{2x}$ when x is large and positive. Find the equation of the curve. [4]

- 7 With respect to the origin O , the position vectors of the points U , V and W are \mathbf{u} , \mathbf{v} and \mathbf{w} respectively. The mid-points of the sides VW , WU and UV of the triangle UVW are M , N and P respectively.

- (i) Show that $\overrightarrow{UM} = \frac{1}{2}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$. [2]

- (ii) Verify that the point G with position vector $\frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$ lies on UM , and deduce that the lines UM , VN and WP intersect at G . [5]

- (iii) Write down, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line through G which is perpendicular to the plane UVW . (It is not necessary to simplify the expression for \mathbf{b} .) [2]

- (iv) It is now given that $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Find the perpendicular distance from O to the plane UVW . [3]

- 8 The set M of matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a , b , c and d are real and $ad - bc = 1$, forms a group (M, \times) under

matrix multiplication. R denotes the set of all matrices $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$.

- (i) Prove that (R, \times) is a subgroup of (M, \times) . [6]

- (ii) By considering geometrical transformations in the x - y plane, find a subgroup of (R, \times) of order 6. Give the elements of this subgroup in exact numerical form. [5]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.



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