General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Thursday 15 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) The polynomial $\mathrm{p}(x)$ is defined by $\mathrm{p}(x) = 6x^3 - 19x^2 + 9x + 10$.

(i) Find $\mathrm{p}(2)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $(2x + 1)$ is a factor of $\mathrm{p}(x)$. *(3 marks)*

(iii) Write $\mathrm{p}(x)$ as the product of three linear factors. *(2 marks)*

(b) Hence simplify $\dfrac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$. *(2 marks)*

2 (a) Obtain the binomial expansion of $(1 - x)^{-3}$ up to and including the term in x^2. *(2 marks)*

(b) Hence obtain the binomial expansion of $\left(1 - \dfrac{5}{2}x\right)^{-3}$ up to and including the term in x^2. *(2 marks)*

(c) Find the range of values of x for which the binomial expansion of $\left(1 - \dfrac{5}{2}x\right)^{-3}$ would be valid. *(2 marks)*

(d) Given that x is small, show that $\left(\dfrac{4}{2 - 5x}\right)^3 \approx a + bx + cx^2$, where a, b and c are integers. *(2 marks)*

3 (a) Given that $\dfrac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}$ can be written in the form $3 + \dfrac{A}{3x - 1} + \dfrac{B}{x - 1}$, where A and B are integers, find the values of A and B. *(4 marks)*

(b) Hence, or otherwise, find $\displaystyle\int \frac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}\,\mathrm{d}x$. *(4 marks)*

4 (a) (i) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(ii) Express $\cos 2x$ in terms of $\cos x$. *(1 mark)*

(b) Show that

$$\sin 2x - \tan x = \tan x \cos 2x$$

for all values of x. *(3 marks)*

(c) Solve the equation $\sin 2x - \tan x = 0$, giving all solutions in degrees in the interval $0° < x < 360°$. *(4 marks)*

5 A curve is defined by the equation

$$y^2 - xy + 3x^2 - 5 = 0$$

(a) Find the y-coordinates of the two points on the curve where $x = 1$. *(3 marks)*

(b) (i) Show that $\dfrac{dy}{dx} = \dfrac{y - 6x}{2y - x}$. *(6 marks)*

(ii) Find the gradient of the curve at each of the points where $x = 1$. *(2 marks)*

(iii) Show that, at the two stationary points on the curve, $33x^2 - 5 = 0$. *(3 marks)*

6 The points A and B have coordinates $(2, 4, 1)$ and $(3, 2, -1)$ respectively. The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$, where O is the origin.

(a) Find the vectors:

(i) $\overrightarrow{OC}$; *(1 mark)*

(ii) $\overrightarrow{AB}$. *(2 marks)*

(b) (i) Show that the distance between the points A and C is 5. *(2 marks)*

(ii) Find the size of angle BAC, giving your answer to the nearest degree. *(4 marks)*

(c) The point $P(\alpha, \beta, \gamma)$ is such that BP is perpendicular to AC.

Show that $4\alpha - 3\gamma = 15$. *(3 marks)*

Turn over for the next question

Turn over ▶

7 Solve the differential equation

$$\frac{dy}{dx} = 6xy^2$$

given that $y = 1$ when $x = 2$. Give your answer in the form $y = f(x)$. *(6 marks)*

8 A disease is spreading through a colony of rabbits. There are 5000 rabbits in the colony. At time t hours, x is the number of rabbits infected. The rate of increase of the number of rabbits infected is proportional to the product of the number of rabbits infected and the number not yet infected.

(a) (i) Formulate a differential equation for $\frac{dx}{dt}$ in terms of the variables x and t and a constant of proportionality k. *(2 marks)*

(ii) Initially, 1000 rabbits are infected and the disease is spreading at a rate of 200 rabbits per hour. Find the value of the constant k.

(You are **not** required to solve your differential equation.) *(2 marks)*

(b) The solution of the differential equation in this model is

$$t = 4\ln\left(\frac{4x}{5000 - x}\right)$$

(i) Find the time after which 2500 rabbits will be infected, giving your answer in hours to one decimal place. *(2 marks)*

(ii) Find, according to this model, the number of rabbits infected after 30 hours. *(4 marks)*

END OF QUESTIONS