Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics FP1R
(6667/01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme		Marks
1.	$\text{f}(z) = 2z^3 - 3z^2 + 8z + 5$		
	$1-2\text{i}$ (is also a root)	seen	B1
	$(z-(1+2\text{i}))(z-(1-2\text{i})) = z^2 - 2z + 5$	Attempt to expand $(z-(1+2\text{i}))(z-(1-2\text{i}))$ or any valid method to establish the quadratic factor e.g. $z = 1 \pm 2\text{i} \Rightarrow z - 1 = \pm 2\text{i} \Rightarrow z^2 - 2z + 1 = -4$ $z = 1 \pm \sqrt{-4} = \dfrac{2 \pm \sqrt{-16}}{2} \Rightarrow b = -2, c = 5$ Sum of roots 2, product of roots 5 $\therefore z^2 - 2z + 5$	M1A1
	$\text{f}(z) = \left(z^2 - 2z + 5\right)\left(2z + 1\right)$	Attempt at **linear** factor with their *cd* in $(z^2 + az + c)(2z + d) = \pm 5$ Or $(z^2 - 2z + 5)(2z + a) \Rightarrow 5a = 5$	M1
	$(z_3) = -\dfrac{1}{2}$		A1
			(5)
			Total 5

Question Number	Scheme		Marks
2.	$f(x) = 3\cos 2x + x - 2$		
(a)	f(2) = -1.9609...... f(3) = 3.8805......	Attempts to evaluate both $f(2)$ and $f(3)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $f(x)$ is continuous) therefore a **root** α is between $x = 2$ and $x = 3$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -1.96.. < 0 < 3.88..) and conclusion.	A1
			(2)
(b)	$\dfrac{\alpha - 2}{"1.9609..."} = \dfrac{3 - \alpha}{"3.8805..."}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(3). Can be implied by working below.	M1
	If any "negative lengths" are used, score M0		
	$(3.88... + 1.96....)\alpha = 3 \times 1.96 + 2 \times 3.88$		
	$\alpha_2 = \dfrac{3 \times 1.96.. + 2 \times 3.88..}{1.96... + 3.88...}$	Follow through their values if seen explicitly.	A1ft
	$\alpha_2 = 2.336$	cao	A1
			(3)
(c)	f(0) = +(1) **or** f(-1) = -(4.248)	Award for correct sign, can be in a table.	B1
	f(-0.5) (= -0.879.....)	Attempt f(-0.5)	M1
	f(-0.25) (= 0.382....)	Attempt **f(-0.25)**	M1
	$\therefore\ -0.5 < \beta < -0.25$	oe with no numerical errors seen	A1
			(4)
			Total 9

Question Number	Scheme		Marks
3.(i)(a)	Rotation of 45 degrees anticlockwise, about the origin	B1: Rotation about (0, 0) B1: 45 degrees (anticlockwise) -45 or clockwise award B0	B1B1
			(2)
(b)	$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	Correct matrix	B1
			(1)
(ii)	$\frac{224}{16}(=14)$	Correct area scale factor. Allow ± 14	B1
	$\det \mathbf{M} = 3\times 3 - k\times -2 = 14$	Attempt determinant and set equal to their area scale factor	M1
		Accept $\det \mathbf{M} = 3\times 3 \pm 2k$ only	
	$k = 2.5$	oe	A1
			(3)
			Total 6

Question Number	Scheme		Marks
4.(a)	$z = \frac{p+2\text{i}}{3+p\text{i}} \cdot \frac{3-p\text{i}}{3-p\text{i}}$	Multiplying top and bottom by Conjugate	M1
	$= \frac{3p - p^2\text{i} + 6\text{i} + 2p}{9+p^2}$	At least 3 correct terms in the numerator, evidence that $\text{i}^2 = -1$and denominator real.	M1
	$= \frac{5p}{p^2+9}, \qquad + \frac{6-p^2}{p^2+9}\text{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(4)
(b)	$\arg(z) = \arctan\left(\frac{\frac{6-p^2}{p^2+9}}{\frac{5p}{p^2+9}}\right)$	Correct method for the argument. Can be implied by correct equation for p	M1
	$\frac{6-p^2}{5p} = 1$	Their $\arg(z)$ in terms of $p = 1$	M1
	$p^2 + 5p - 6 = 0$	Correct 3TQ	A1
	$(p+6)(p-1) = 0 \Rightarrow x =$	M1:Attempt to solve their quadratic in p	M1
	$p = 1, p = -6$	A1:**both**	A1
			(5)
			Total 9
(a) **Way 2**	$a + b\text{i} = \frac{p+2\text{i}}{3+p\text{i}}$	Equate to $a+b\text{i}$ then rearrange and equate real and imaginary parts.	M1
	$3a - pb = p,\ ap + 3b = 2$	Two equations for a and b in terms of p and attempt to solve for a and b in terms of p	dM1
	$= \frac{5p}{p^2+9}, \qquad + \frac{6-p^2}{p^2+9}\text{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(5)

Question Number	Scheme		Marks
5.(a)	$r(r^2-3)=r^3-3r$	r^3-3r	B1
	$\sum_{r=1}^{n} r(r^2-3)=\sum_{r=1}^{n} r^3-3\sum_{r=1}^{n} r$		
	$=\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$	M1: An attempt to use at least one of the standard formulae correctly. A1: Correct expression	M1A1
	$=\frac{1}{4}n(n+1)(n(n+1)-6)$	Attempt factor of $\frac{1}{4}n(n+1)$ before given answer	M1
	$=\frac{1}{4}n(n+1)(n^2+n-6)$		
	$=\frac{1}{4}n(n+1)(n+3)(n-2)$	cso	A1
			(5)
(b)	$\sum_{r=10}^{50} r(r^2-3)=\mathrm{f}(50)-\mathrm{f}(9\,or\,10)$	Require some use of the result in part (a) for method.	M1
	$=\frac{1}{4}(50)(51)(53)(48)-\frac{1}{4}(9)(10)(12)(7)$	Correct expression	A1
	= 1621800 - 1890		
	= 1619910	cao	A1
			(3)
			Total 8

Question Number	Scheme		Marks
6.(a)	$\mathbf{A}+\mathbf{B}=\begin{pmatrix}1&2\\-1&1\end{pmatrix}$	M1:Correct attempt at matrix addition with 3 elements correct A1: Correct matrix	M1A1
	$2\mathbf{A}-\mathbf{B}=\begin{pmatrix}5&1\\-2&-1\end{pmatrix}$	M1: Correct attempt to double **A** and subtract **B** 3 elements correct A1: Correct matrix	M1A1
	$(\mathbf{A}+\mathbf{B})(2\mathbf{A}-\mathbf{B})=\begin{pmatrix}1&2\\-1&1\end{pmatrix}\begin{pmatrix}5&1\\-2&-1\end{pmatrix}$		
	$\begin{pmatrix}1&2\\-1&1\end{pmatrix}\begin{pmatrix}5&1\\-2&-1\end{pmatrix}=\begin{pmatrix}1&-1\\-7&-2\end{pmatrix}$	M1: Correct method to multiply A1: cao	M1A1
			(6)
(a) Way 2	$(\mathbf{A}+\mathbf{B})(2\mathbf{A}-\mathbf{B})=2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2$	M1: Expands brackets with at least 3 correct terms A1: Correct expansion	M1A1
	$\mathbf{A}^2=\begin{pmatrix}3&2\\-2&-1\end{pmatrix},\mathbf{BA}=\begin{pmatrix}-3&-1\\-1&0\end{pmatrix},$ $\mathbf{AB}=\begin{pmatrix}-2&3\\1&-1\end{pmatrix},\mathbf{B}^2=\begin{pmatrix}1&0\\0&1\end{pmatrix}$	M1: Attempts $\mathbf{A}^2$, $\mathbf{B}^2$ and **AB** or **BA** A1: Correct matrices	M1A1
	$2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2=\begin{pmatrix}1&-1\\-7&-2\end{pmatrix}$	M1: Substitutes into their expansion A1: Correct matrix	M1A1
(b)	$\mathbf{MC}=\mathbf{A}\Rightarrow\mathbf{C}=\mathbf{M}^{-1}\mathbf{A}$	May be implied by later work	B1
	$\mathbf{M}^{-1}=\frac{1}{-2-7}\begin{pmatrix}-2&1\\7&1\end{pmatrix}$	An attempt at their $\frac{1}{\det\mathbf{M}}\begin{pmatrix}-2&1\\7&1\end{pmatrix}$	M1
	$\mathbf{C}=\frac{1}{-2-7}\begin{pmatrix}-2&1\\7&1\end{pmatrix}\begin{pmatrix}2&1\\-1&0\end{pmatrix}$	Correct order required and an attempt to multiply	dM1
	$\mathbf{C}=-\frac{1}{9}\begin{pmatrix}-5&-2\\13&7\end{pmatrix}$	oe	A1
			(4)
			Total 10
(b) Way 2	$\begin{pmatrix}1&-1\\-7&-2\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}2&1\\-1&0\end{pmatrix}$	Correct statement	B1
	$a-c=2,\ b-d=1$ $-7a-2c=-1,\ -7b-2d=0$	Multiplies correctly to obtain 4 equations	M1
	$a=\frac{5}{9},b=\frac{2}{9},c=-\frac{13}{9},d=-\frac{7}{9}$	M1: Solves to obtain values for a, b, c and d A1: Correct values	M1A1

Question Number	Scheme		Marks
7.(a)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$ or $\frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = 2a.\frac{1}{2ap}$	$\frac{dy}{dx} = k\,x^{-\frac{1}{2}}$ or $ky\frac{dy}{dx} = c$ their $\frac{dy}{dp} \times \left(\frac{1}{\text{their}\,\frac{dx}{dp}}\right)$	M1
	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ *or* $2y\frac{dy}{dx} = 4a$ *or* $\frac{dy}{dx} = 2a.\frac{1}{2ap}$	Correct differentiation	A1
	At *P*, gradient of normal = *-p*	Correct normal gradient with no errors seen.	A1
	$y - 2ap = -p(x - ap^2)$	Applies $y - 2ap = \text{their } m_N\left(x - ap^2\right)$ or $y = \left(\text{their } m_N\right)x + c$ using $x = ap^2$ *and* $y = 2ap$ in an attempt to find c. **Their m_N must be different from their m_T and must be a function of *p*.**	M1
	$y + px = 2ap + ap^3$ *	cso **given answer**	A1*
			(5)
(b)	$y - px = -2ap - ap^3$	oe	B1
			(1)
(c)	$y = 0 \Rightarrow x = 2a + ap^2$	M1: y = 0 in either normal or solves simultaneously to find x A1: y = 0 and correct x coordinate.	M1A1
			(2)
(d)	*S* is (a, 0)	Can be implied below	B1
	Area *SPQP'* = $\frac{1}{2} \times \left(\text{"}2a + ap^2\text{"} - a\right) \times 2ap \times 2$	Correct method for the area of the quadrilateral.	M1
	$= 2a^2 p\left(1 + p^2\right)$	Any equivalent form	A1
			(3)
			Total 11

Question Number	Scheme		Marks
8.			
	$\frac{12}{7}c = -\frac{1}{t^2} \times -\frac{6}{7}c + \frac{2c}{t}$	Substitutes $\left(-\frac{6}{7}c,\ \frac{12}{7}c\right)$ into the equation of the tangent	M1
	$\frac{12}{7}c = -\frac{1}{t^2} \times -\frac{6}{7}c + \frac{2c}{t} \Rightarrow$ $6t^2 - 7t - 3 = 0$	Correct 3TQ in terms of t	A1
	$6t^2 - 7t - 3 = 0 \Rightarrow (3t+1)(2t-3) = 0 \Rightarrow t =$	Attempt to solve their 3TQ for t	M1
	$t = -\frac{1}{3},\ t = \frac{3}{2} \Rightarrow \left(-\frac{1}{3}c,\ -3c\right),\ \left(\frac{3}{2}c,\ \frac{2}{3}c\right)$	M1: Uses at least one of their values of t to find A or B. A1: Correct coordinates.	M1A1
			(5)
			Total 5

Question Number	Scheme		Marks
9.(a)	When n = 1, rhs = lhs = 2		B1
	Assume true for $n = k$ so $\sum_{r=1}^{k}(r+1)2^{r-1} = k2^k$		
	$\sum_{r=1}^{k+1}(r+1)2^{r-1} = k2^k + (k+1+1)2^{k+1-1}$	M1: Attempt to add (k + 1)th term A1: Correct expression	M1A1
	$= k2^k + (k+2)2^k$		
	$= 2\times k2^k + 2\times 2^k$		
	$=(k+1)2^{k+1}$	At least one correct intermediate step required.	A1
	If the result is **true for *n* = *k*** then it has been shown **true for *n* = *k* + 1.** As it is **true for *n* = 1** then it is **true for all *n*** (positive integers.)	cso, statements can be seen anywhere in the solution.	A1
		Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0	
			(5)
(b)	When $n = 1$ $u_1 = 4^2 - 2^4 = 0$	$4^2 - 2^4 = 0$ seen	B1
	When $n = 2$ $u_2 = 4^3 - 2^5 = 32$	$4^3 - 2^5 = 32$ seen	B1
	True for $n = 1$ and $n = 2$		
	Assume $u_k = 4^{k+1} - 2^{k+3}$ and $u_{k+1} = 4^{k+2} - 2^{k+4}$		
	$u_{k+2} = 6u_{k+1} - 8u_k$ $= 6\left(4^{k+2} - 2^{k+4}\right) - 8\left(4^{k+1} - 2^{k+3}\right)$	M1: Attempts u_{k+2} in terms of u_{k+1} and u_k A1: Correct expression	M1A1
	$= 6.4^{k+2} - 6.2^{k+4} - 8.4^{k+1} + 8.2^{k+3}$		
	$= 6.4^{k+2} - 3.2^{k+5} - 2.4^{k+2} + 2.2^{k+5}$	Attempt u_{k+2} in terms of 4^{k+2} and 2^{k+5}	M1
	$= 4.4^{k+2} - 2^{k+5} = 4^{k+3} - 2^{k+5}$		
	So $u_{k+2} = 4^{(k+2)+1} - 2^{(k+2)+3}$	Correct expression	A1
	If the result **is true for *n* = *k*** and ***n* = *k* + 1** then it has been shown **true for *n* = *k* + 2**. As it is **true for *n* = 1 and *n* = 2** then it is **true for all *n*** (positive integers.)	cso, statements can be seen anywhere in the solution.	A1
		Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0	
			(7)
			Total 12

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