General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MFP3

Unit Further Pure 3

Friday 11 June 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x + 3 + \sin y$$

and $$y(1) = 1$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(3 marks)*

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2 (a) Find the value of the constant k for which $k \sin 2x$ is a particular integral of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y = \sin 2x$$ *(3 marks)*

(b) Hence find the general solution of this differential equation. *(4 marks)*

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3 (a) Explain why $\int_1^{\infty} 4x\mathrm{e}^{-4x}\,\mathrm{d}x$ is an improper integral. *(1 mark)*

(b) Find $\int 4x\mathrm{e}^{-4x}\,\mathrm{d}x$. *(3 marks)*

(c) Hence evaluate $\int_1^{\infty} 4x\mathrm{e}^{-4x}\,\mathrm{d}x$, showing the limiting process used. *(3 marks)*

4 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + \frac{3}{x}y = (x^4 + 3)^{\frac{3}{2}}$$

given that $y = \frac{1}{5}$ when $x = 1$. *(9 marks)*

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5 (a) Write down the expansion of $\cos 4x$ in ascending powers of x up to and including the term in x^4. Give your answer in its simplest form. *(2 marks)*

(b) (i) Given that $y = \ln(2 - e^x)$, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

(You may leave your expression for $\frac{d^3y}{dx^3}$ unsimplified.) *(6 marks)*

(ii) Hence, by using Maclaurin's theorem, show that the first three non-zero terms in the expansion, in ascending powers of x, of $\ln(2 - e^x)$ are

$$-x - x^2 - x^3$$ *(2 marks)*

(c) Find

$$\lim_{x \to 0} \left[\frac{x \ln(2 - e^x)}{1 - \cos 4x} \right]$$ *(3 marks)*

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6 The polar equation of a curve C_1 is

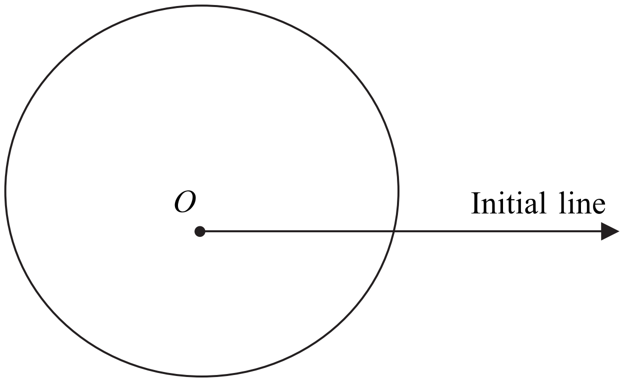
$$r = 2(\cos\theta - \sin\theta), \quad 0 \leqslant \theta \leqslant 2\pi$$

(a) (i) Find the cartesian equation of C_1. *(4 marks)*

(ii) Deduce that C_1 is a circle and find its radius and the cartesian coordinates of its centre. *(3 marks)*

(b) The diagram shows the curve C_2 with polar equation

$$r = 4 + \sin\theta, \quad 0 \leqslant \theta \leqslant 2\pi$$



(i) Find the area of the region that is bounded by C_2. *(6 marks)*

(ii) Prove that the curves C_1 and C_2 do not intersect. *(4 marks)*

(iii) Find the area of the region that is outside C_1 but inside C_2. *(2 marks)*

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7 (a) Given that $x = t^{\frac{1}{2}}$, $x > 0$, $t > 0$ and y is a function of x, show that:

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2t^{\frac{1}{2}}\dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(2 marks)*

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4t\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}t}$. *(3 marks)*

(b) Hence show that the substitution $x = t^{\frac{1}{2}}$ transforms the differential equation

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - (8x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x} + 12x^3y = 12x^5$$

into

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 4\frac{\mathrm{d}y}{\mathrm{d}t} + 3y = 3t$$ *(2 marks)*

(c) Hence find the general solution of the differential equation

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - (8x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x} + 12x^3y = 12x^5$$

giving your answer in the form $y = \mathrm{f}(x)$. *(7 marks)*

END OF QUESTIONS