

2. A uniform square lamina S has side $2a$. The radius of gyration of S about an axis through a vertex, perpendicular to S , is k .

(a) Show that $k^2 = \frac{8a^2}{3}$. (4)

The lamina S is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to S and passes through a vertex.

(b) By writing down an equation of rotational motion for S , find the period of small oscillations of S about its position of stable equilibrium. (5)



3. A raindrop falls vertically under gravity through a stationary cloud. At time $t = 0$, the raindrop is at rest and has mass m_0 . As the raindrop falls, water condenses onto it from the cloud so that the mass of the raindrop increases at a constant rate c . At time t , the mass of the raindrop is m and the speed of the raindrop is v . The resistance to the motion of the raindrop has magnitude mkv , where k is a constant. Show that

$$\frac{dv}{dt} + v \left(k + \frac{c}{m_0 + ct} \right) = g \tag{7}$$



5.

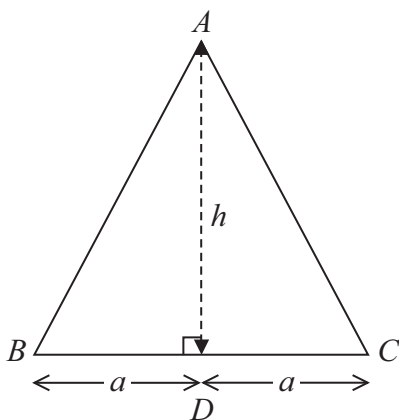


Figure 1

A uniform triangular lamina ABC , of mass M , has $AB = AC$ and $BC = 2a$. The mid-point of BC is D and $AD = h$, as shown in Figure 1.

Show, using integration, that the moment of inertia of the lamina about an axis through A , perpendicular to the plane of the lamina, is

$$\frac{M}{6} (a^2 + 3h^2)$$

[You may assume without proof that the moment of inertia of a uniform rod, of length $2l$ and mass m , about an axis through its midpoint and perpendicular to the rod, is $\frac{1}{3}ml^2$.]

(10)



