General Certificate of Education
Advanced Level Examination
January 2012

# Mathematics MM2B

## Unit Mechanics 2B

## Wednesday 25 January 2012 1.30 pm to 3.00 pm

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**1** A plane is dropping packets of aid as it flies over a flooded village. The speed of a packet when it leaves the plane is $60\,\mathrm{m\,s^{-1}}$. The packet has mass 25 kg.

The packet falls a vertical distance of 34 metres to reach the ground.

**(a)** Calculate the kinetic energy of the packet when it leaves the plane. *(2 marks)*

**(b)** Calculate the potential energy lost by the packet as it falls to the ground. *(2 marks)*

**(c)** Assume that the effect of air resistance on the packet as it falls can be neglected.

**(i)** Find the kinetic energy of the packet when it reaches the ground. *(2 marks)*

**(ii)** Hence find the speed of the packet when it reaches the ground. *(2 marks)*

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**2** A particle, of mass 50 kg, moves on a smooth horizontal plane. A single horizontal force

$$[(300t - 60t^2)\,\mathbf{i} + 100\mathrm{e}^{-2t}\,\mathbf{j}] \text{ newtons}$$

acts on the particle at time $t$ seconds.

The vectors **i** and **j** are perpendicular unit vectors.

**(a)** Find the acceleration of the particle at time $t$. *(2 marks)*

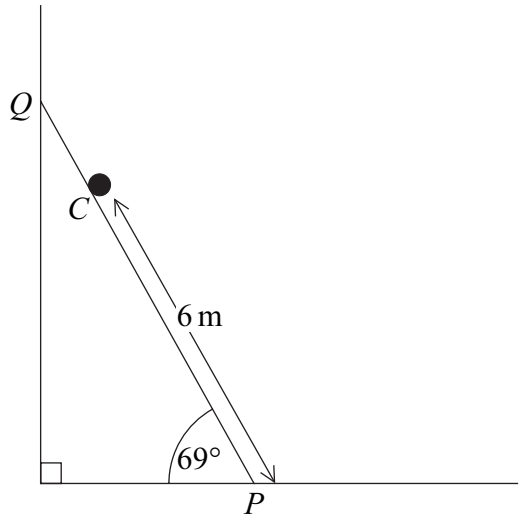
**(b)** When $t = 0$, the velocity of the particle is $(7\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find the velocity of the particle at time $t$. *(4 marks)*

**(c)** Calculate the speed of the particle when $t = 1$. *(4 marks)*

**3** A uniform ladder $PQ$, of length 8 metres and mass 28 kg, rests in equilibrium with its foot, $P$, on a rough horizontal floor and its top, $Q$, leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall and the angle between the ladder and the floor is 69°.

A man, of mass 72 kg, is standing at the point $C$ on the ladder so that the distance $PC$ is 6 metres. The man may be modelled as a particle at $C$.




**(a)** Draw a diagram to show the forces acting on the ladder. *(2 marks)*

**(b)** With the man standing at the point $C$, the ladder is on the point of slipping.

**(i)** Show that the magnitude of the reaction between the ladder and the vertical wall is 256 N, correct to three significant figures. *(4 marks)*

**(ii)** Find the coefficient of friction between the ladder and the horizontal floor. *(4 marks)*

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**4** A car travels along a straight horizontal road. When its speed is $v\,\mathrm{m\,s^{-1}}$, the car experiences a resistance force of magnitude $25v$ newtons.

**(a)** The car has a maximum constant speed of $42\,\mathrm{m\,s^{-1}}$ on this road.

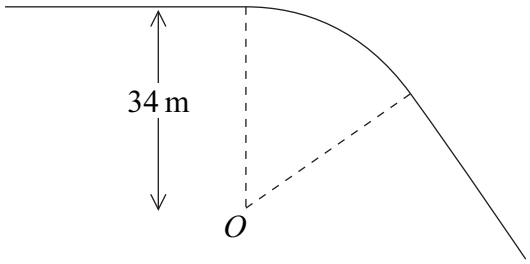
Show that the power being used to propel the car at this speed is 44 100 watts. *(2 marks)*

**(b)** The car has mass 1500 kg.

Find the acceleration of the car when it is travelling at $15\,\mathrm{m\,s^{-1}}$ on this road under a power of 44 100 watts. *(4 marks)*

**5** A parcel is placed on a flat rough horizontal surface in a van. The van is travelling along a horizontal road. It travels around a bend of radius 34 m at a constant speed. The coefficient of friction between the parcel and the horizontal surface in the van is 0.85 .

Model the parcel as a particle travelling around part of a circle of radius 34 m and centre $O$, as shown in the diagram.




Find the greatest speed at which the van can travel around the bend without causing the parcel to slide. *(6 marks)*

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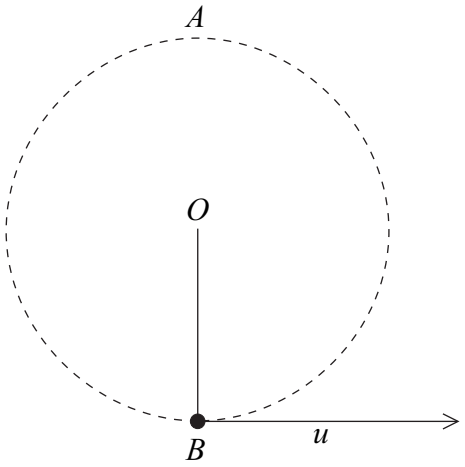
**6** Alice places a toy, of mass 0.4 kg, on a slope. The toy is set in motion with an initial velocity of $1\,\text{m}\,\text{s}^{-1}$ down the slope. The resultant force acting on the toy is $(2 - 4v)$ newtons, where $v\,\text{m}\,\text{s}^{-1}$ is the toy's velocity at time $t$ seconds after it is set in motion.

**(a)** Show that $\dfrac{\text{d}v}{\text{d}t} = -10\,(v - 0.5)$. *(2 marks)*

**(b)** By using $\displaystyle\int \frac{1}{v - 0.5}\,\text{d}v = -\int 10\,\text{d}t$, find $v$ in terms of $t$. *(5 marks)*

**(c)** Find the time taken for the toy's velocity to reduce to $0.55\,\text{m}\,\text{s}^{-1}$. *(3 marks)*

**7** A small bead, of mass $m$, is suspended from a fixed point $O$ by a light inextensible string of length $a$. With the string taut, the bead is at the point $B$, vertically below $O$, when it is set into vertical circular motion with an initial horizontal velocity $u$, as shown in the diagram.




The string does not become slack in the subsequent motion. The velocity of the bead at the point $A$, where $A$ is vertically above $O$, is $v$.

**(a)** Show that $v^2 = u^2 - 4ag$. *(2 marks)*

**(b)** The ratio of the tensions in the string when the bead is at the two points $A$ and $B$ is $2:5$.

**(i)** Find $u$ in terms of $g$ and $a$. *(7 marks)*

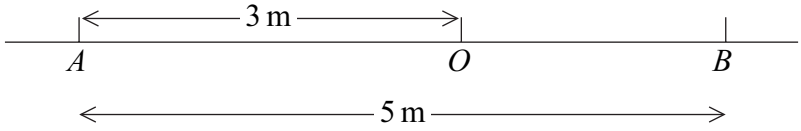
**(ii)** Find the ratio $u:v$. *(2 marks)*

**8** An elastic string has one end attached to a point $O$ fixed on a rough horizontal surface. The other end of the string is attached to a particle of mass 2 kg. The elastic string has natural length 0.8 metres and modulus of elasticity 32 newtons.

The particle is pulled so that it is at the point $A$, on the surface, 3 metres from the point $O$.

**(a)** Calculate the elastic potential energy when the particle is at the point $A$. *(3 marks)*

**(b)** The particle is released from rest at the point $A$ and moves in a straight line towards $O$. The particle is next at rest at the point $B$. The distance $AB$ is 5 metres.




Find the frictional force acting on the particle as it moves along the surface. *(6 marks)*

**(c)** Show that the particle does not remain at rest at the point $B$. *(2 marks)*

**(d)** The particle next comes to rest at a point $C$ with the string slack.

Find the distance $BC$. *(2 marks)*

**(e)** Hence, or otherwise, find the total distance travelled by the particle after it is released from the point $A$. *(1 mark)*