General Certificate of Education
January 2007
Advanced Level Examination

**MATHEMATICS**                    **MFP2**
**Unit Further Pure 2**

AQA

ASSESSMENT and
QUALIFICATIONS
ALLIANCE

Thursday 1 February 2007   9.00 am to 10.30 am

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**For this paper you must have:**
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

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Time allowed: 1 hour 30 minutes

**Instructions**
- Use blue or black ink or ball-point pen.  Pencil should only be used for drawing.
- Write the information required on the front of your answer book.  The *Examining Body* for this paper is AQA.  The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

**MFP2**

Answer **all** questions.

**1** (a) Given that

$$4\cosh^2 x = 7\sinh x + 1$$

find the two possible values of $\sinh x$. *(4 marks)*

(b) Hence obtain the two possible values of $x$, giving your answers in the form $\ln p$.
*(3 marks)*

**2** (a) Sketch on one diagram:

(i) the locus of points satisfying $|z - 4 + 2i| = 2$; *(3 marks)*

(ii) the locus of points satisfying $|z| = |z - 3 - 2i|$. *(3 marks)*

(b) Shade on your sketch the region in which

both $$|z - 4 + 2i| \leqslant 2$$

and $$|z| \leqslant |z - 3 - 2i|$$ *(2 marks)*

**3** The cubic equation

$$z^3 + 2(1 - i)z^2 + 32(1 + i) = 0$$

has roots $\alpha$, $\beta$ and $\gamma$.

(a) It is given that $\alpha$ is of the form $k$i, where $k$ is real. By substituting $z = k$i into the equation, show that $k = 4$. *(5 marks)*
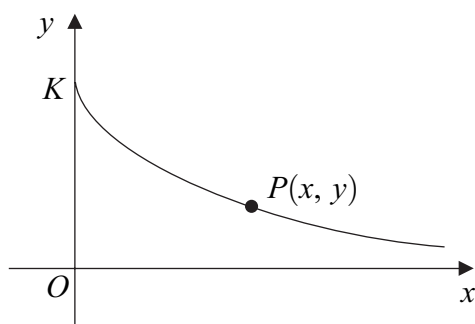
(b) Given that $\beta = -4$, find the value of $\gamma$. *(2 marks)*

**4** (a) Given that $y = \operatorname{sech} t$, show that:

   (i) $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\operatorname{sech} t \tanh t$; *(3 marks)*

   (ii) $\left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \operatorname{sech}^2 t - \operatorname{sech}^4 t$. *(2 marks)*

   (b) The diagram shows a sketch of part of the curve given parametrically by

$$x = t - \tanh t \qquad y = \operatorname{sech} t$$



The curve meets the $y$-axis at the point $K$, and $P(x, y)$ is a general point on the curve. The arc length $KP$ is denoted by $s$. Show that:

   (i) $\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \tanh^2 t$; *(4 marks)*

   (ii) $s = \ln \cosh t$; *(3 marks)*

   (iii) $y = \mathrm{e}^{-s}$. *(2 marks)*

   (c) The arc $KP$ is rotated through $2\pi$ radians about the $x$-axis. Show that the surface area generated is

$$2\pi(1 - \mathrm{e}^{-s}) \qquad \textit{(4 marks)}$$

**Turn over for the next question**

**5** (a) Prove *by induction* that, if $n$ is a positive integer,

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \qquad \textit{(5 marks)}$$

(b) Find the value of $\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)^6$. *(2 marks)*

(c) Show that

$$(\cos\theta + i\sin\theta)(1 + \cos\theta - i\sin\theta) = 1 + \cos\theta + i\sin\theta \qquad \textit{(3 marks)}$$

(d) Hence show that

$$\left(1 + \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)^6 + \left(1 + \cos\dfrac{\pi}{6} - i\sin\dfrac{\pi}{6}\right)^6 = 0 \qquad \textit{(4 marks)}$$

**6** (a) Find the three roots of $z^3 = 1$, giving the non-real roots in the form $e^{i\theta}$, where $-\pi < \theta \leqslant \pi$. *(2 marks)*

(b) Given that $\omega$ is one of the non-real roots of $z^3 = 1$, show that

$$1 + \omega + \omega^2 = 0 \qquad \textit{(2 marks)}$$

(c) By using the result in part (b), or otherwise, show that:

(i) $\dfrac{\omega}{\omega + 1} = -\dfrac{1}{\omega}$; *(2 marks)*

(ii) $\dfrac{\omega^2}{\omega^2 + 1} = -\omega$; *(1 mark)*

(iii) $\left(\dfrac{\omega}{\omega + 1}\right)^k + \left(\dfrac{\omega^2}{\omega^2 + 1}\right)^k = (-1)^k\, 2\cos\dfrac{2}{3}k\pi$, where $k$ is an integer. *(5 marks)*

**7** (a) Use the identity $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$ with $A = (r + 1)x$ and $B = rx$ to show that

$$\tan rx \tan(r + 1)x = \frac{\tan(r + 1)x}{\tan x} - \frac{\tan rx}{\tan x} - 1 \qquad \textit{(4 marks)}$$

(b) Use the method of differences to show that

$$\tan\frac{\pi}{50}\tan\frac{2\pi}{50} + \tan\frac{2\pi}{50}\tan\frac{3\pi}{50} + ... + \tan\frac{19\pi}{50}\tan\frac{20\pi}{50} = \frac{\tan\dfrac{2\pi}{5}}{\tan\dfrac{\pi}{50}} - 20 \qquad \textit{(5 marks)}$$

**END OF QUESTIONS**

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