RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4727/01

MATHEMATICS

Further Pure Mathematics 3

THURSDAY 25 JANUARY 2007

Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 **(i)** Show that the set of numbers $\{3, 5, 7\}$, under multiplication modulo 8, does not form a group. [2]

(ii) The set of numbers $\{3, 5, 7, a\}$, under multiplication modulo 8, forms a group. Write down the value of a. [1]

(iii) State, justifying your answer, whether or not the group in part **(ii)** is isomorphic to the multiplicative group $\{e, r, r^2, r^3\}$, where e is the identity and $r^4 = e$. [2]

2 Find the equation of the line of intersection of the planes with equations

$$\mathbf{r}.(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 4 \qquad \text{and} \qquad \mathbf{r}.(\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}) = 6,$$

giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [5]

3 **(i)** Solve the equation $z^2 - 6z + 36 = 0$, and give your answers in the form $r(\cos\theta \pm \mathrm{i}\sin\theta)$, where $r > 0$ and $0 \leqslant \theta \leqslant \pi$. [4]

(ii) Given that Z is either of the roots found in part **(i)**, deduce the exact value of Z^{-3}. [3]

4 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2 - y^2}{xy}. \qquad \text{(A)}$$

(i) Use the substitution $y = xz$, where z is a function of x, to obtain the differential equation

$$x\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{1 - 2z^2}{z}.$$ [3]

(ii) Hence show by integration that the general solution of the differential equation (A) may be expressed in the form $x^2(x^2 - 2y^2) = k$, where k is a constant. [6]

5 A multiplicative group G of order 9 has distinct elements p and q, both of which have order 3. The group is commutative, the identity element is e, and it is given that $q \neq p^2$.

(i) Write down the elements of a proper subgroup of G

(a) which does not contain q, [1]

(b) which does not contain p. [1]

(ii) Find the order of each of the elements pq and pq^2, justifying your answers. [3]

(iii) State the possible order(s) of proper subgroups of G. [1]

(iv) Find two proper subgroups of G which are distinct from those in part **(i)**, simplifying the elements. [4]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} + 3y = 2x + 1.$$

Find

(i) the complementary function, [1]

(ii) the general solution. [5]

In a particular case, it is given that $\frac{dy}{dx} = 0$ when $x = 0$.

(iii) Find the solution of the differential equation in this case. [3]

(iv) Write down the function to which y approximates when x is large and positive. [1]

7 The position vectors of the points A, B, C, D, G are given by

$$\mathbf{a} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \quad \mathbf{c} = \mathbf{i} + 5\mathbf{j} + 4\mathbf{k}, \quad \mathbf{d} = 3\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}, \quad \mathbf{g} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$$

respectively.

(i) The line through A and G meets the plane BCD at M. Write down the vector equation of the line through A and G and hence show that the position vector of M is $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$. [6]

(ii) Find the value of the ratio $AG : AM$. [1]

(iii) Find the position vector of the point P on the line through C and G, such that $\overrightarrow{CP} = \frac{4}{3}\overrightarrow{CG}$. [2]

(iv) Verify that P lies in the plane ABD. [4]

8 **(i)** Use de Moivre's theorem to find an expression for $\tan 4\theta$ in terms of $\tan \theta$. [4]

(ii) Deduce that $\cot 4\theta = \dfrac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta}$. [1]

(iii) Hence show that one of the roots of the equation $x^2 - 6x + 1 = 0$ is $\cot^2\left(\frac{1}{8}\pi\right)$. [3]

(iv) Hence find the value of $\operatorname{cosec}^2\left(\frac{1}{8}\pi\right) + \operatorname{cosec}^2\left(\frac{3}{8}\pi\right)$, justifying your answer. [5]