Version 1.0

AQA

General Certificate of Education
June 2010

Mathematics   **MM2B**

Mechanics 2B

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.
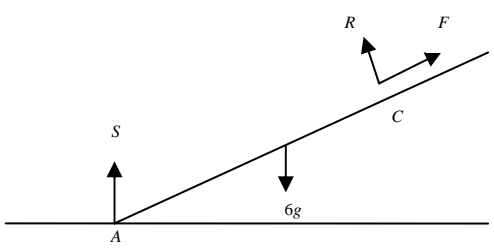
MM2B

Q	Solution	Marks	Total	Comments
1	$v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$	M1		M1 for either $\dfrac{\mathrm{d}s}{\mathrm{d}t}$ or 1 of 2 terms correct (ignore signs)
	$= 10t - 12 \sin 4t$	A1A1	3	
	Total		**3**	
2(a)	Kinetic energy $= \dfrac{1}{2} \times 3 \times 4^2$	M1		
	$= 24$ (J)	A1	2	
(b)	PE lost is			
	$= 3 \times g \times 51$	M1		
	$= 153g$ or 1499.4			
	$= 1500$ J	A1	2	Accept 1499, 153g
(c)(i)	KE is $24 + 153g$	M1		M1 '(a)' + '(b)'
	$= 1523.4$			
	$= 1520$ J	A1		(if done (c)(i) in (b) 0 marks; if done (b) and then (c)(i) in (b) M1 only)
(ii)	Using KE $= \dfrac{1}{2}mv^2$			
	$v^2 = 1015.6$	M1		
	Speed of stone is 31.9 ms^{-1}	A1	4	Accept 31.8 from 1520
				If use constant acceleration formulae in 2D, possible 4 marks in (c) BUT **no marks** if initial speed is treated as being vertical
(d)	eg Stone is a particle	E1	1	
	No air resistance			Not no resistance; accept no wind resistance
	Total		**9**	
3(a)	Symmetry	E1	1	Only accept 'symmetry'
(b)	Moments about B:			
	$0.4 \times 4 + 0.1 \times 8 = 0.5 \times \bar{x}$	M1A1		M1 3 terms , 2 correct
	$\bar{x} = \dfrac{2.4}{0.5}$			
	$= 4.8$ cm	A1	3	
	Total		**4**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	Using $\mathbf{F} = m\mathbf{a}$,			
	$400 \cos \frac{\pi}{2} t\, \mathbf{i} + 600t^2\, \mathbf{j} = 200\, \mathbf{a}$	M1		
	$\mathbf{a} = 2 \cos \frac{\pi}{2} t\, \mathbf{i} + 3t^2\, \mathbf{j}$	A1	2	
(b)	$\mathbf{v} = \int a \, \mathrm{d}t$	M1		M1 for either $\int a \, \mathrm{d}t$ or 1 of 2 terms correct
	$= \frac{4}{\pi} \sin \frac{\pi}{2} t\, \mathbf{i} + t^3 \mathbf{j} + \mathbf{c}$	A1m1		m1 for $+ \mathbf{c}$
	When $t = 4$, $\mathbf{r} = -3\mathbf{i} + 56\mathbf{j}$, $64\mathbf{j} + \mathbf{c} = -3\mathbf{i} + 56\mathbf{j}$	m1		
	$\therefore \mathbf{c} = -3\mathbf{i} - 8\mathbf{j}$ $\therefore \mathbf{v} = (\frac{4}{\pi} \sin \frac{\pi}{2} t -3)\mathbf{i} + (t^3 - 8)\mathbf{j}$	A1	5	Do not accept $\frac{2}{\frac{\pi}{2}}$ Accept 1.27 for $\frac{4}{\pi}$
(c)	When particle is moving due west, northerly component is zero	M1		
	$\therefore t^3 - 8 = 0$	A1✓		
	$t = 2$	A1	3	
(d)	When $t = 2$, $\mathbf{v} = -3\mathbf{i} + 0\mathbf{j}$	B1✓		
	Speed of particle is 3 m s^{-1}	B1	2	B1 for change -3 to $+3$
	Total		**12**	
5	$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\lambda}{v^{\frac{1}{4}}}$	M1		
	$\int v^{\frac{1}{4}} \mathrm{d}v = -\int \lambda \, \mathrm{d}t$	m1		Condone one of $v^{-\frac{1}{4}}$, $+\int \lambda \mathrm{d}t$, $\frac{1}{\lambda}$
	$\frac{4}{5} v^{\frac{5}{4}} = -\lambda t + c$	A1A1 m1		m1 for $+ c$
	$t = 0$, $v = u$ $\therefore c = \frac{4}{5} u^{\frac{5}{4}}$	A1		
	$\therefore v^{\frac{5}{4}} = u^{\frac{5}{4}} - \frac{5}{4} \lambda t$			
	$v = \left(u^{\frac{5}{4}} - \frac{5}{4} \lambda t \right)^{\frac{4}{5}}$	A1	7	
	Total		**7**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	Using power = force × velocity Power = (30 × 48) × 48 $\quad$ = 69120 watts	M1 A1	2	AG
(b)	When speed is 40 m s^{-1}, max force exerted is $\dfrac{69120}{40}$ = 1728 N Accelerating force is '1728' – 1200 N Using $F = ma$: 528 = 1200a a = 0.44 m s^{-2}	 B1 M1 m1 A1	4	
(c)	Force exerted by engine is $\dfrac{69120}{v}$ Force exerted by the engine $= 30v - mg\sin 3$ $30v - 615.47$ (or $1200g\sin 3$)$= \dfrac{69120}{v}$ $30v^2 - 615.47v - 69120 = 0$ $v = \dfrac{615.47 \pm \sqrt{615.47^2 + 4 \times 30 \times 69120}}{2 \times 30}$ Speed is 59.3 m s^{-1}	B1 M1 A1A1 A1 M1 A1	 7	(Use of cos3 delete A1,A1 of 3 A terms) A2 All terms correct A1 Two terms correct SC3 for $30v^2 + 615.47v - 69120 = 0$
	Total		**13**	
7(a)		B2	2	B1 for S and $6g$ (in correct place) B1 for R and F or combined vertical force at C
(b)	Moments about C: $3 \times S \times \cos 20 = 6g \times 1 \times \cos 20$ S = 19.6 N or $2g$	M1A1 A1	3	M1 2 terms, 1 term correct R, F not correct 0 marks in (c)(i) and (c)(ii)
(c)(i)	Moments about A: $2 \times 6g \times \cos 20 = R \times 3$ R = 36.8 N (or resolving, $R = 6g\cos20 - S\cos20$ $\quad\quad\quad = 4g\cos20$)	M1A1 A1		**Or** Moments about mid-point of rod: $2 \times S \times \cos 20 = P \times 1 \times \cos 20$ P = 39.2 N or $4g$ (Or resolving vertically $P = 4g$)
(ii)	Resolve parallel to AB: $S\cos 70 + F = 6g\cos 70$ $F = 4g\cos 70$ $\quad = 13.4$ N (or $F = 6g\sin20 - S\sin20 = 4g\sin20$)	 M1 A1	 5	$R = P \times \cos 20$ $\quad\quad$ M1 A1 $\quad = 36.8$ N $\quad\quad\quad\quad$ A1 $F = P \times \sin 20$ $\quad\quad\quad$ M1 $\quad = 13.4$ N $\quad\quad\quad\quad$ A1
(d)	Using $F = \mu R$: $13.4 = \mu \times 36.8$ $\mu = 0.364$ or $\tan20$	M1 A1√	 2	M1 '(c)(ii)' $= \mu$ '(c)(i)' (condone $\geq$)
	Total		**12**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	Using conservation of energy: $\frac{1}{2}mv^2 = 3mg(1-\cos\theta)$	M1A1		M1 $\frac{1}{2}mv^2 = mgh$
	$v^2 = 6g(1-\cos15)$	m1		
	$v = (6g[1-\cos15])^{\frac{1}{2}}$ $= 1.42$	A1	4	SC3: 1.41
(b)	When particle is at rest,			
	resolve radially $T = mg\cos15$	M1A1		M1 $T - mg\cos15 = \dfrac{mv^2}{r}$ or $T = mg\sin15$
	$22 = mg\cos15$ $m = 2.32$	A1	3	
	Total		**7**	
9	As particle moves, $T = \dfrac{mv^2}{r}$	M1		**or** using unknown as extension:
	If radius is r, extension is $r-1.2$	B1		If extension is x, radius is $1.2+x$ B1
	Using $T = \dfrac{\lambda x}{l}$:			Using $T = \dfrac{\lambda x}{l}$:
	$T = \dfrac{192(r-1.2)}{1.2}$	M1		$T = \dfrac{192x}{1.2}$ M1
	$= 160(r-1.2)$	A1		$= 160x$ A1
	$T = \dfrac{mv^2}{r} \Rightarrow 160(r-1.2) = \dfrac{8\times3^2}{r}$	M1		$T = \dfrac{mv^2}{r} \Rightarrow 160x = \dfrac{8\times3^2}{1.2+x}$ M1 M1
	$160r^2 - 192r = 72$ (or $192r^2 - 230.4r = 86.4$)	A1		$192x + 160x^2 = 72$ A1
	$20r^2 - 24r - 9 = 0$ $(10r+3)(2r-3) = 0$	M1		$20x^2 + 24x - 9 = 0$ $(10x-3)(2x+3) = 0$ M1
	$r = 1.5$ or -0.3 Radius is 1.5	A1	8	$x = 0.3$ or -1.5 Radius is 1.5 A1
	Total		**8**	
	TOTAL		**75**	