

Mark Scheme 4721

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1	(i)	$\frac{21-3}{4-1} = \frac{18}{3} = 6$	M1	2	Uses $\frac{y_2 - y_1}{x_2 - x_1}$ 6 (not left as $\frac{18}{3}$)
			A1		
	(ii)	$\frac{dy}{dx} = 2x + 1$ $2 \times 3 + 1 = 7$	B1	2	
			B1		
2	(i)	$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$	M1	2	$\frac{1}{27^{\frac{2}{3}}}$ or $27^{\frac{2}{3}} = 9$ or 3^{-2} soi $\frac{1}{9}$
			A1		
	(ii)	$5\sqrt{5} = 5^{\frac{3}{2}}$	B1	1	
	(iii)	$\frac{1-\sqrt{5}}{3+\sqrt{5}} = \frac{(1-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$ $= \frac{8-4\sqrt{5}}{4}$ $= 2-\sqrt{5}$	M1	3	Multiply numerator and denominator by conjugate $(\sqrt{5})^2 = 5$ soi $2-\sqrt{5}$
			B1		
			A1		
3	(i)	$2x^2 + 12x + 13 = 2(x^2 + 6x) + 13$ $= 2[(x+3)^2 - 9] + 13$ $= 2(x+3)^2 - 5$	B1 B1 M1	4	$a = 2$ $b = 3$ $13 - 2b^2$ or $13 - b^2$ or $\frac{13}{2} - b^2$ (their b) $c = -5$
			A1		
	(ii)	$2(x+3)^2 - 5 = 0$ $(x+3)^2 = \frac{5}{2}$ $x = -3 \pm \sqrt{\frac{5}{2}}$	M1	3	Uses correct quadratic formula or completing square method $x = \frac{-12 \pm \sqrt{40}}{4}$ or $(x+3)^2 = \frac{5}{2}$ $x = -3 \pm \sqrt{\frac{5}{2}}$ or $-3 \pm \frac{1}{2}\sqrt{10}$
			A1		
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4	<p>(i)</p> $(x-4)(x-3)(x+1)$ $\equiv (x^2 - 7x + 12)(x+1)$ $\equiv x^3 + x^2 - 7x^2 - 7x + 12x + 12$ $\equiv x^3 - 6x^2 + 5x + 12$ <p>(ii)</p> <p>(iii)</p>		<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1√</p>	<p>$x^2 - 7x + 12$ or $x^2 - 2x - 3$ or $x^2 - 3x - 4$ seen</p> <p>3</p> <p>Attempt to multiply a quadratic by a linear factor or attempt to list an 8 term expansion of all 3 brackets</p> <p>$x^3 - 6x^2 + 5x + 12$ (AG) obtained (no wrong working seen)</p> <p>+ve cubic with 3 roots (not 3 line segments)</p> <p>(0, 12) labelled or indicated on y-axis</p> <p>3</p> <p>(-1, 0), (3,0), (4, 0) labelled or indicated on x-axis</p> <p>2</p> <p>Reflect <i>their</i> (ii) in either x- or y-axis</p> <p>Reflect <i>their</i> (ii) in x-axis</p>
5	<p>(i)</p> $1 < 4x - 9 < 5$ $10 < 4x < 14$ $2.5 < x < 3.5$ <p>(ii)</p> $y^2 \geq 4y + 5$ $y^2 - 4y - 5 \geq 0$ $(y-5)(y+1) \geq 0$ $y \leq -1, y \geq 5$	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>2 equations or inequalities both dealing with all 3 terms</p> <p>2.5 and 3.5 seen oe</p> <p>3</p> <p>$2.5 < x < 3.5$ (or '$x > 2.5$ <u>and</u> $x < 3.5$')</p> <p>$y^2 - 4y - 5 = 0$ soi</p> <p>Correct method to solve quadratic</p> <p>-1, 5</p> <p>(SR If both values obtained from trial and improvement, award B3)</p> <p>Correct method to solve inequality</p> <p>5</p> <p>$y \leq -1, y \geq 5$</p>	

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6	(i) $x^4 - 10x^2 + 25 = 0$ Let $y = x^2$ $y^2 - 10y + 25 = 0$ $(y-5)^2 = 0$ $y = 5$ $x^2 = 5$ $x = \pm\sqrt{5}$	*M1 dep*M1 A1 A1	4	Use a substitution to obtain a quadratic or $(x^2 - 5)(x^2 - 5) = 0$ Correct method to solve a quadratic 5 (not $x = 5$ with no subsequent working) $x = \pm\sqrt{5}$	
	(ii)	$y = \frac{2x^5}{5} - \frac{20x^3}{3} + 50x + 3$ $\frac{dy}{dx} = 2x^4 - 20x^2 + 50$	B1 B1	2	$2x^4$ or $-20x^2$ oe seen $2x^4 - 20x^2 + 50$ (integers required)
	(iii)	$2x^4 - 20x^2 + 50 = 0$ $x^4 - 10x^2 + 25 = 0$ which has 2 roots	M1 A1	2	<i>their</i> $\frac{dy}{dx} = 0$ seen (or implied by correct answer) 2 stationary points www in any part
7	(i) $y = x^2 - 5x + 4$ $y = x - 1$ $x^2 - 5x + 4 = x - 1$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $x = 1 \quad x = 5$ $y = 0 \quad y = 4$	M1 M1 A1 A1	4	Substitute to find an equation in x (or y) Correct method to solve quadratic $x = 1, 5$ $y = 0, 4$ (N.B. This final A1 may be awarded in part (ii) if y coordinates only seen in part (ii)) SR one correct (x,y) pair www B1	
	(ii)	2 points of intersection	B1	1	
	(iii)	EITHER $x^2 - 5x + 4 = x + c$ has 1 solution $x^2 - 6x + (4 - c) = 0$ $b^2 - 4ac = 0$ $36 - 4(4 - c) = 0$ $c = -5$ OR $\frac{dy}{dx} = 1 = 2x - 5$ $x = 3 \quad y = -2$ $-2 = 3 + c$ $c = -5$	M1 M1 A1 A1 M1 A1 A1	4	$x^2 - 5x + 4 = x + c$ has 1 soln seen or implied Discriminant = 0 or $(x - a)^2 = 0$ soi $36 - 4(4 - c) = 0$ or $9 = 4 - c$ $c = -5$ Algebraic expression for gradient of curve = non-zero gradient of line used $2x - 5 = 1$ $x = 3$ $c = -5$ SR $c = -5$ without any working B1

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8	(i)	$\text{Height of box} = \frac{8}{x^2}$ $4 \text{ vertical faces} = 4 \times \frac{8}{x}$ $= \frac{32}{x}$ $\text{Total surface area} = x^2 + x^2 + \frac{32}{x}$ $A = 2x^2 + \frac{32}{x}$	<p>*B1</p> <p>*B1</p> <p>B1 dep on both **</p>	$\text{Area of 1 vertical face} = \frac{8}{x^2} \times x$ $= \frac{8}{x}$ <p>Correct final expression</p>	3
	(ii)	$\frac{dA}{dx} = 4x - \frac{32}{x^2}$	<p>B1</p> <p>B1</p> <p>B1</p>	$4x$ kx^2 $-32x^{-2}$	3
	(iii)	$4x - \frac{32}{x^2} = 0$ $4x^3 = 32$ $x = 2$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	$\frac{dA}{dx} = 0 \text{ soi}$ $x = 2$ <p>Check for minimum</p> <p>Correctly justified</p> <p>SR If $x = 2$ stated www but with no evidence of differentiated expression(s) having been used in part (iii) B1</p>	4

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9	(i)	$\left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$ $(7, 2)$	M1	2	Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
			A1		(7, 2) (integers required)
	(ii)	$\sqrt{(7-4)^2 + (2-(-2))^2}$ $= \sqrt{3^2 + 4^2}$ $= 5$	M1	2	Uses $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
			A1		5
(iii)	$(x-7)^2 + (y-2)^2 = 25$	B1√	3	$(x-7)^2$ and $(y-2)^2$ used (<i>their</i> centre)	
		B1√		$r^2 = 25$ used (<i>their</i> r^2)	
		B1		$(x-7)^2 + (y-2)^2 = 25$ cao	
				<p><u>Expanded form:</u></p> $-14x$ and $-4y$ used B1√	
				$r = \sqrt{g^2 + f^2 - c}$ used B1√	
				$x^2 + y^2 - 14x - 4y + 28 = 0$ B1 cao	
				<p><u>By using ends of diameter:</u></p> $(x-4)(x-10) + (y+2)(y-6) = 0$ Both x brackets correct B1 Both y brackets correct B1 Final equation fully correct B1	
(iv)	$\text{Gradient of } AB = \frac{6-(-2)}{10-4} = \frac{4}{3}$	B1	5	oe	
	$\text{Gradient of tangent} = -\frac{3}{4}$	B1√			
	$y-(-2) = -\frac{3}{4}(x-4)$	M1		Correct equation of straight line through A, any non-zero gradient	
	$3x + 4y = 4$	A1		a, b, c need not be integers	