4737 Decision Mathematics 2

1	**(i)**	*Stage* / *State* / *Action* / *Working* / *Maximin* *0* / *0* / *10* / ***10*** *1* / *0* / *11* / ***11*** *1* / *1* / *0* / *14* / ***14*** *2* / *0* / *15* / ***15*** *0* / *0* / *min(12, **10**)=**10*** *2* / *min(10, **14**)=**10*** / ***10*** *0* / *min(13, **10**)=**10*** *2* / *1* / *1* / *min(10, **11**)=**10*** *2* / *min(11, **14**)=**11*** / ***11*** *1* / *min(9, **11**)=**9*** *2* / *2* / *min(10, **14**)=**10*** / ***10*** *3* / *min(7, **15**)=**7*** *3* / *1* / *min(8, **11**)=**8*** *3* / *min(12, **15**)=**12*** / ***12*** *0* / *min(15, **10**)=**10*** *3* / *0* / *1* / *min(14, **11**)=**11*** *2* / *min(16, **10**)=**10*** *3* / *min(13, **12**)=**12*** / ***12***	M1 M1 M1 A1 M1 A1	**Answered on insert** Transferring maximin values from stage 1 correctly Completing working column for stage 2 (method) Calculating maximin values for stage 2 (method) Maximin values correct for stage 2 (cao) Transferring maximin values from stage 2 correctly Working column for stage 3 correct (cao)	**[6]**
	(ii)	Maximin value = **12** Maximin route = **(0; 0) – (1; 3) – (2; 3) – (3; 0)**	B1 M1 A1	12 (cao) Route, or in reverse, follow through their table if possible, condone omission of (0; 0) Correct route, including (0; 0) (cao)	**[3]**
					Total = 9

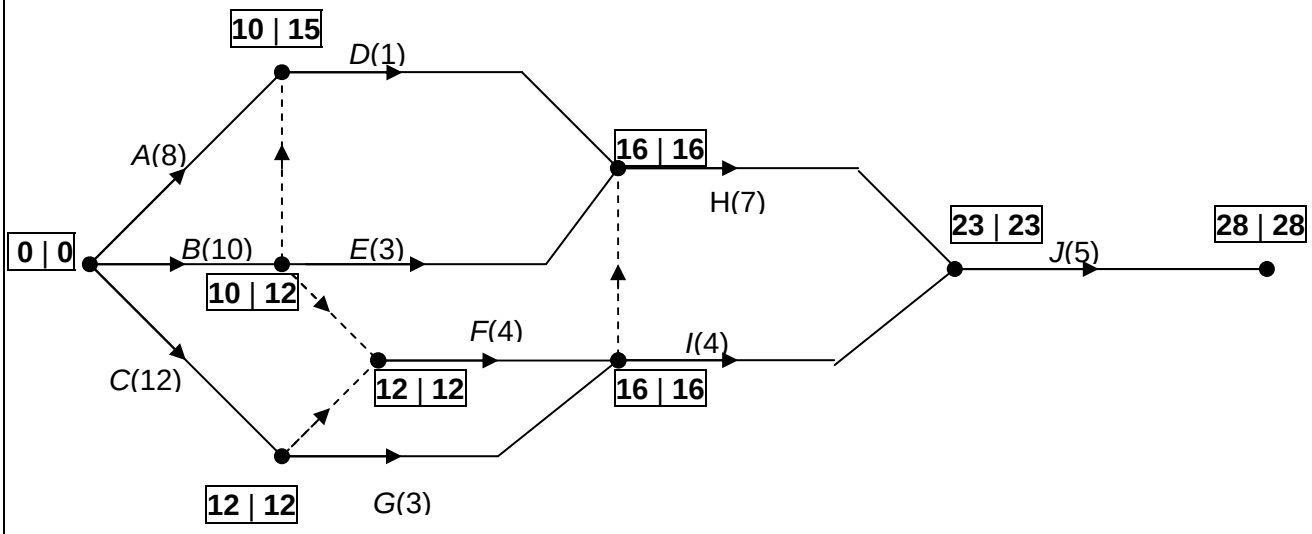
2 (i)

Activity	*Duration (days)*	*Immediate predecessors*
A	*8*	-
B	*10*	-
C	*12*	-
D	*1*	**A B**
E	*3*	**B**
F	*4*	**B C**
G	*3*	**C**
H	*7*	**D E F G**
I	*4*	**F G**
J	*5*	**H I**

	Answered on insert
B1	Precedences correct for *D* and *E*
B1	Precedences correct for *F* and *G*
B1	Precedences correct for *H, I* and *J*

[3]

(ii)



M1	Substantially correct attempt at forward pass (at most one independent error)
M1	Substantially correct attempt at backward pass (at most one independent error) No follow through, 28 given in question
A1	Both passes wholly correct
B1	*C F H J* and no others (no follow through)

Critical activities ***C F H J*** **[4]**

(iii)

A	B	C	D	E	F	G	H	I	J
1	***1***	***3***	***2***	***1***	***1***	***2***	***2***	***3***	***4***

B1	*J* correct
B1	*H* and *I* correct
B1	*F* and *G* correct
B1	*D* and *E* correct
B1	*B* and *C* correct
B1	*A* correct

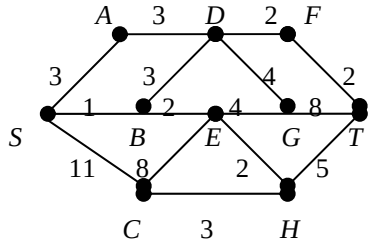
[6]

(iv)

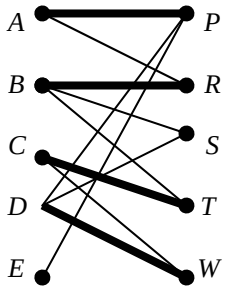
Minimum delay **1** day	B1 1
Maximum delay **3** days	B1 3

[2]

Total = 15

3	**(i)**	4+3-2+8-2+7 = **18** litres per second	M1 A1	**Answered on insert** Imply method mark from 18, 20 or 22 cao	**[2]**
	(ii)	3 litres per second flow out of *B* (arc *BD*) so only 2 litres per second can enter *B* from *E* and only 1 litre per second can enter *B* from *S*. At least 4 litres per second flow out of *E* to *G*, 2 litres per second from *E* to *B* and 2 litres per second from *E* to *H*, so 8 litres per second must flow into *E* from *C*. 8 litres per second flows from *C* to *E* and at most 11 litres per second enters *C* from *S*, so at most 3 litres per second flows from *C* to *H*. Also, 2 litres per second flow from *E* to *H* so the most that can enter *H* is 5 litres per second. But at least 5 litres per second leave *H* along *HT*, hence the flow in *HT* is 5 litres per second.	B1 B1 M1 A1	At B: 3 out and 1 + 2 in At *E*: (at least) 4 + 2 + 2 out Considering *C* to show flow in *CH* is <u>at most</u> 3 Must explicitly refer to ≤ 3, or $2 \leq$ flow ≤ 3, not just stating 3 At *H*: 2 + 3 in	**[4]**
	(iii)	 Flow augmenting route: ***S A D F T*** or ***S A D G T*** Cut: $X = \{S, B\}$, $Y = \{A, C, D, E, F, G, H, T\}$ Or $X = \{S, A, B\}$, $Y = \{C, D, E, F, G, H, T\}$	M1 A1 B1 B1	Substantially correct attempt (at least 12 correct) (Not shown as excess capacities and potential backflows) All correct (cao) Either of these (correct) flow augmenting routes Either of these (correct) cuts described in any way, or marked clearly on diagram	**[4]**
	(iv)	*B* would have at most 3 litres per second entering it and at least 5 litres per second leaving.	M1 A1	Identifying that problem is at *B* A correct explanation	**[2]**
				Total =	**12**

4 (i)



B1 Bipartite graph correct

B1 Incomplete matching correct (clearly shown, or shown on a separate bipartite graph)

[2]

(ii) ***E – P – A – R – B – S***

Anya = restaurant review
Ben = sports news
Connie = theatre review
Derek = weather report
Emma = problem page

M1 A valid alternating path from *E* to *S*, written out
A1 This path written out (not just shown on diagram)

B1 *A = R B = S C = T D = W E = P* (cao)

[3]

(iii) Add a dummy column

	P	R	S	T	W	X
J	*56*	*56*	*51*	*57*	*58*	*60*
K	*53*	*52*	*53*	*54*	*54*	*60*
L	*57*	*55*	*52*	*58*	*60*	*60*
M	*59*	*55*	*53*	*59*	*57*	*60*
N	*57*	*57*	*53*	*59*	*60*	*60*
O	*58*	*56*	*51*	*56*	*57*	*60*

B1 Adding a dummy column of equal ‘costs’ of at least 60 minutes

Reduce rows

5	*5*	*0*	*6*	*7*	*9*
1	*0*	*1*	*2*	*2*	*8*
5	*3*	*0*	*6*	*8*	*8*
6	*2*	*0*	*6*	*4*	*7*
4	*4*	*0*	*6*	*7*	*7*
7	*5*	*0*	*5*	*6*	*9*

M1 Substantially correct attempt at reducing rows (at most one error)

Then reduce columns

4	*5*	*0*	*4*	*5*	*2*
0	*0*	*1*	*0*	*0*	*1*
4	*3*	*0*	*4*	*6*	*1*
5	*2*	*0*	*4*	*2*	*0*
3	*4*	*0*	*4*	*5*	*0*
6	*5*	*0*	*3*	*4*	*2*

M1 Substantially correct attempt at reducing columns (at most one error)

A1 Correct reduced cost matrix, with rows reduced first (cao)

[4]

Cross out 0's using 3 (minimum number of) lines

4	5	**0**	4	5	**2**
0	**0**	**1**	**0**	**0**	**1**
4	3	**0**	4	6	**1**
5	2	**0**	4	2	**0**
3	4	**0**	4	5	**0**
6	5	**0**	3	4	**2**

Augment by 2

2	3	0	2	3	2
0	0	3	0	0	3
2	1	0	2	4	1
3	0	0	2	0	0
1	2	0	2	3	0
4	3	0	1	2	2

M1 Follow through their reduced cost matrix for crossing through 0's and augmenting (without errors)

A1 Augment by 2 in a single augmentation (cao)

Cross out 0's using 4 (minimum number of) lines

2	3	**0**	2	3	**2**
0	**0**	**3**	**0**	**0**	**3**
2	1	**0**	2	4	**1**
3	**0**	**0**	**2**	**0**	**0**
1	2	**0**	2	3	**0**
4	3	**0**	1	2	**2**

Augment by 1

1	2	0	1	2	2
0	0	4	0	0	4
1	0	0	1	3	1
3	0	1	2	0	1
0	1	0	1	2	0
3	2	0	0	1	2

Alternative

2	3	**0**	2	3	2
0	**0**	**3**	**0**	**0**	**3**
2	1	**0**	2	4	1
3	**0**	**0**	**2**	**0**	**0**
1	**2**	**0**	**2**	**3**	**0**
4	3	**0**	1	2	2

1	2	0	1	2	1
0	0	4	0	0	3
1	0	0	1	3	0
3	0	1	2	0	0
1	2	1	2	3	0
3	2	0	0	1	1

M1 Follow through their matrix for crossing through 0's and augmenting (correct for theirs)

A1 (Either) correct final matrix (cao)

[4]

To get a complete allocation

1	2	**0**	1	2	2
0	0	4	0	0	4
1	**0**	0	1	3	1
3	0	1	2	**0**	1
0	1	0	1	2	**0**
3	2	0	**0**	1	2

1	2	**0**	1	2	1
0	0	4	0	0	3
1	**0**	0	1	3	0
3	0	1	2	**0**	0
1	2	1	2	3	**0**
3	2	0	**0**	1	1

Jeremy Kath Laura Mohammed Ollie
Sports Problems Restaurant Weather Theatre

B1 $J = S \quad K = P \quad L = R \quad M = W \quad O = T$

$51 + 53 + 55 + 57 + 56 = 272$

M1 Correct method

$272 \times £0.25 = £68$

A1 £68 (cao) with units

[3]

Total = 16

5	**(i)**	5 (10 - 4) ÷ 2 = 3	B1 M1 A1	5 3 or 7 3	**[3]**
	(ii)	<table><tr><th></th><th>D</th><th>E</th><th>F</th><th>row min</th></tr><tr><td>S</td><td>0</td><td>4</td><td>-2</td><td>-2</td></tr><tr><td>T</td><td>-4</td><td>2</td><td>-4</td><td>-4</td></tr><tr><td>U</td><td>2</td><td>-6</td><td>0</td><td>-6</td></tr><tr><td>col max</td><td>2</td><td>4</td><td>0</td><td></td></tr></table> Play-safe for rugby club (rows) is Sanjeev Play-safe for cricket club (cols) is Fiona Not stable because -2 ≠ 0	 M1 M1 A1 A1 B1	 Calculating row minima Calculating col maxima (or equivalent) Sanjeev or S (not just -2 or identifying row) Fiona or F (not just 0 or identifying column) Any correct explanation	**[5]**
	(iii)	 Fiona Ursula	 B1 B1	Follow through their play-safe strategies if possible F U	**[2]**
	(iv)	Sanjeev's row dominates Tom's row Doug Fiona's column dominates Doug's (once Tom's row has been removed)	B1 M1 A1	This or any equivalent statement about Tom and Sanjeev (note: Tom is named in the question) Doug This or any equivalent statement about Doug and Fiona	**[3]**
	(v)	*E*: $4p - 6(1-p) = 10p - 6$ *F*: $-2p$ $10p - 6 = -2p$ $p = 0.5$	M1 A1	Follow through their choice from part (iv) Both expressions seen in any form (note: *D* gives $2(1-p) = 2 - 2p$) $p = 0.5$ (cao)	**[2]**
	(vi)	Delete *T* row *0 4 -2* *2 -6 0* Multiply entries by -1 to show scores for Cricket club *0 -4 2* *-2 6 0* Add 4 to make entries non-negative *4 0 6* *2 10 4* Choose Doug with probability *x*, Euan with probability *y* and Fiona with probability *z*. If Sanjeev plays, expected score = $4x + 6z$ If Ursula plays, expected score = $2x + 10y + 4z$	 B1 B1 B1	 Delete *T* row <u>and</u> multiply entries by -1 Add 4 to make entries non-negative Identifying meaning of *x*, *y*, *z* or implied by reference to *S* for $4x + 6z$ and *U* for $2x + 10y + 4z$	**[3]**
	(vii)	$z = \frac{5}{6}$ maximum value for $m = 5$ Hence, maximum value for $M = 1$	M1 A1		**[2]**
					Total = 20