

5. A statistician believes a coin is biased and the probability, p , of getting a head when the coin is tossed is less than 0.5

The statistician decides to test this by tossing the coin 10 times and recording the number, X , of heads. He sets up the hypotheses $H_0 : p = 0.5$ and $H_1 : p < 0.5$ and rejects the null hypothesis if $x < 3$

- (a) Find the size of the test. (1)

- (b) Show that the power function of this test is

$$(1 - p)^8 (36p^2 + 8p + 1) \tag{3}$$

Table 1 gives values, to 2 decimal places, of the power function for the statistician's test.

p	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Power	0.93	0.82	r	0.53	0.38	0.26	s	0.10

Table 1

- (c) Calculate the value of r and the value of s . (2)

Question 5 parts (d) and (e) continue on page 16



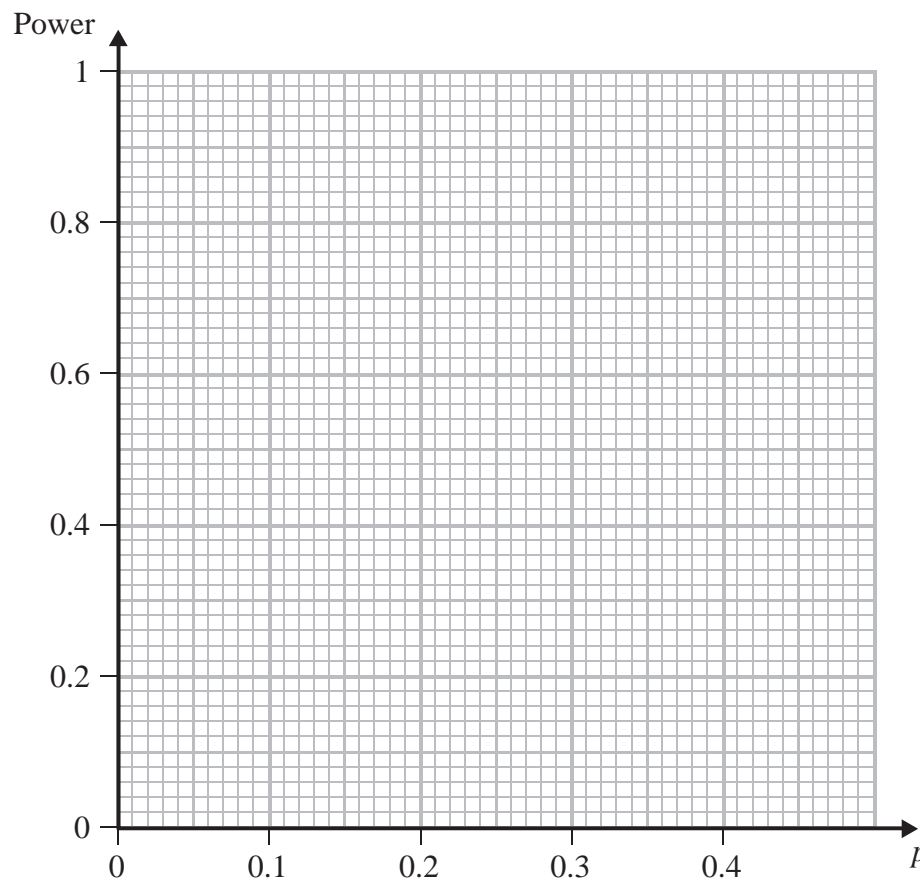
Question 5 continued

For your convenience Table 1 is repeated here.

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Table 1

- (d) On the axes below draw the graph of the power function for the statistician's test. (2)
- (e) Find the range of values of p for which the probability of accepting the coin as unbiased, when in fact it is biased, is less than or equal to 0.4 (3)





6. (a) Explain what is meant by the sampling distribution of an estimator T of the population parameter θ . (1)

(b) Explain what you understand by the statement that T is a biased estimator of θ . (1)

A population has mean μ and variance σ^2

A random sample X_1, X_2, \dots, X_{10} is taken from this population.

(c) Calculate the bias of each of the following estimators of μ .

$$\hat{\mu}_1 = \frac{X_3 + X_5 + X_7}{3}$$

$$\hat{\mu}_2 = \frac{5X_1 + 2X_2 + X_9}{6}$$

$$\hat{\mu}_3 = \frac{3X_{10} - X_1}{3} \quad (4)$$

(d) Find the variance of each of these three estimators. (6)

(e) State, giving a reason, which of these three estimators for μ is

(i) the best estimator,

(ii) the worst estimator. (3)



