

### 4730 Mechanics 3

<p><b>1</b></p>	$0.4(3\cos 60^\circ - 4) = -I \cos \theta \quad (= -1)$ $0.4(3\sin 60^\circ) = I \sin \theta \quad (= 1.03920)$ $[\tan \theta = -1.5\sqrt{3} / (1.5 - 4);$ $I^2 = 0.4^2[(1.5 - 4)^2 + (1.5\sqrt{3})^2]]$ $\theta = 46.1 \text{ or } I = 1.44$ $I = 1.44 \text{ or } \theta = 46.1$	<p>M1 A1 A1</p> <p>M1 A1 M1</p> <p>A1ft [7]</p>	<p>For using <math>I = \Delta mv</math> in one direction</p> <p>SR: Allow B1 (max 1/3) for <math>3\cos 60^\circ - 4 = -I \cos \theta</math> and <math>3\sin 60^\circ = I \sin \theta</math></p> <p>For eliminating I or <math>\theta</math> (allow following SR case)</p> <p>Allow for <math>\theta</math> (only) following SR case.</p> <p>For substituting for <math>\theta</math> or for I (allow following SR case)</p> <p>ft incorrect <math>\theta</math> or I; allow for <math>\theta</math> (only) following SR case.</p>
	<p>Alternatively</p> $I^2 = 1.2^2 + 1.6^2 - 2 \times 1.2 \times 1.6 \cos 60^\circ \quad \text{or}$ $'V'^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \cos 60^\circ$ $I = 1.44$ $\frac{\sin \theta}{3(\text{or } 1.2)} = \frac{\sin 60}{\sqrt{13}(\text{or } 2.08)} \quad \text{or}$ $\frac{\sin \alpha}{4(\text{or } 1.6)} = \frac{\sin 60}{\sqrt{13}(\text{or } 2.08)} \text{ and } \theta = 120 - \alpha$ $\theta = 46.1$	<p>M1 A1 M1 A1 M1</p> <p>A1ft [7]</p>	<p>For use of cosine rule</p> <p>For correct use of factor 0.4 (= m)</p> <p>For use of sine rule</p> <p><math>\alpha</math> must be angle opposite 1.6; (<math>\alpha = 73.9</math>)</p> <p>ft value of I or 'V'</p>
<p><b>2</b></p>	$2a + 3b = 2 \times 4$ $b - a = 0.6 \times 4$ $[2(b - 2.4) + 3b = 8]$ $b = 2.56$ $v = 2.56$	<p>M1 A1 M1 A1 M1 A1 B1ft [7]</p>	<p>For using the principle of conservation of momentum</p> <p>For using NEL</p> <p>For eliminating a</p> <p>ft <math>v = b</math></p>
<p><b>3(i)</b></p>	$2W(a \cos 45^\circ) = T(2a)$ $W = \sqrt{2} T$	<p>M1 A1 A1 [3]</p>	<p>For using 'mmt of <math>2W = \text{mmt of } T'</math></p> <p>AG</p>
<p><b>(ii)</b></p>	<p>Components (H, V) of force on BC at B are  <math>H = -T/\sqrt{2}</math> and <math>V = T/\sqrt{2} - 2W</math></p> $W(a \cos \alpha) + H(2a \sin \alpha) = V(2a \cos \alpha)$ $[W \cos \alpha - T \sqrt{2} \sin \alpha = T \sqrt{2} \cos \alpha - 4W \cos \alpha]$ $T \sqrt{2} \sin \alpha = (5W - T \sqrt{2}) \cos \alpha$ $\tan \alpha = 4$	<p>B1 M1 A1</p> <p>M1 A1ft A1 [6]</p>	<p>For taking moments about C for BC</p> <p>For substituting for H and V and reducing equation to the form <math>X \sin \alpha = Y \cos \alpha</math></p>

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	<p>Alternatively for part (ii)</p> <p>anticlockwise mmt =</p> $W(a \cos\alpha) + 2W(2a \cos\alpha + a \cos 45^\circ)$ $= T[2a \cos(\alpha - 45^\circ) + 2a]$ $[5W \cos\alpha + \sqrt{2} W = T(\sqrt{2} \cos\alpha + \sqrt{2} \sin\alpha) + 2]$ $T \sqrt{2} \sin \alpha = (5W - T \sqrt{2}) \cos \alpha$ $\tan \alpha = 4$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[6]</p>	<p>For taking moments about C for the whole</p> <p>For reducing equation to the form</p> $X \sin \alpha = Y \cos \alpha$
4(i)	$[-0.2(v + v^2) = 0.2a]$ $[v \, dv/dx = -(v + v^2)]$ $[1/(1 + v)] \, dv/dx = -1$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For using Newton's second law</p> <p>For using <math>a = v \, dv/dx</math></p> <p>AG</p>
(ii)	$\ln(1 + v) = -x (+ C)$ $\ln(1 + v) = -x + \ln 3$ $[(1 + dx/dt)/3 = e^{-x} \rightarrow dx/dt = 3e^{-x} - 1]$ $\rightarrow e^x \, dx/dt = 3 - e^x]$ $[-e^x/(3 - e^x)] \, dx/dt = -1$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For integrating</p> <p>For transposing for v and using <math>v = dx/dt</math></p> <p>AG</p>
(iii)	$[\ln(3 - e^x) = -t + \ln 2]$ $\ln(3 - e^x) = -t + \ln 2$ <p>Value of t is 1.96 (or <math>\ln\{2 \div (3 - e)\}</math>)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>For integrating and using <math>x(0) = 0</math></p>
5(i)	<p>Loss of EE = <math>120(0.5^2 - 0.3^2)/(2 \times 1.6)</math></p> <p>and gain in PE = <math>1.5 \times 4</math></p> <p><math>v = 0</math> at B and loss of EE = gain in PE (= 6)</p> <p><math>\rightarrow</math> distance AB is 4m</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For using <math>EE = \lambda x^2/2L</math> and <math>PE = Wh</math></p> <p>For comparing EE loss and PE gain</p> <p>AG</p>
(ii)	$[120e/1.6 = 1.5]$ <p><math>e = 0.02</math></p> <p>Loss of EE = <math>120(0.5^2 - 0.02^2)/(2 \times 1.6)</math></p> <p>(or <math>120(0.3^2 - 0.02^2)/(2 \times 1.6)</math>)</p> <p>Gain in PE = <math>1.5(2.1 - 1.6 - 0.02)</math></p> <p>(or <math>1.5(1.9 + 1.6 + 0.02)</math> loss)</p> $[KE \text{ at max speed} = 9.36 - 0.72]$ <p>(or <math>3.36 + 5.28</math>)</p> $\frac{1}{2} (1.5/9.8)v^2 = 9.36 - 0.72$ <p>Maximum speed is <math>10.6 \text{ ms}^{-1}</math></p>	<p>M1</p> <p>A1</p> <p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>For using <math>T = mg</math> and <math>T = \lambda x/L</math></p> <p>ft incorrect e only</p> <p>ft incorrect e only</p> <p>For using KE at max speed</p> <p>= Loss of EE - Gain (or + loss) in PE</p>
	<p>First alternative for (ii)</p> <p>x is distance AP</p> $[\frac{1}{2} (1.5/9.8)v^2 + 1.5x + 120(0.5 - x)^2/3.2 = 120 \times 0.5^2/3.2]$ <p>KE and PE terms correct</p> <p>EE terms correct</p> $v^2 = 470.4x - 490x^2$ $[470.4 - 980x = 0]$ <p><math>x = 0.48</math></p> <p>Maximum speed is <math>10.6 \text{ ms}^{-1}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>For using energy at P = energy at A</p> <p>For attempting to solve <math>dv^2/dx = 0</math></p>

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	<p>Second alternative for (ii)  <math>[120e/1.6 = 1.5]</math>  <math>e = 0.02</math>  <math>[1.5 - 120(0.02 + x)/1.6 = 1.5 \ddot{x}/g]</math></p> <p><math>n = \sqrt{490}</math>  <math>a = 0.48</math>                  Maximum speed is <math>10.6 \text{ ms}^{-1}</math></p>	<p>M1                  A1                  M1                    M1                  A1                    A1                  A1</p>	<p>For using <math>T = mg</math> and <math>T = \lambda x/L</math></p> <p>For using Newton's second law                  For obtaining the equation in the form  <math>\ddot{x} = -n^2x</math>, using <math>(AB - L - e_{\text{equil}})</math> for                  amplitude and using <math>v_{\text{max}} = na</math>.</p>
6(i)	<p>PE gain by P = <math>0.4g \times 0.8 \sin \theta</math>                  PE loss by Q = <math>0.58g \times 0.8 \theta</math></p> <p><math>\frac{1}{2}(0.4 + 0.58)v^2 = g \times 0.8(0.58 \theta - 0.4 \sin \theta)</math>  <math>v^2 = 9.28 \theta - 6.4 \sin \theta</math></p>	<p>B1                  B1                  M1                  A1ft                  A1                  [5]</p>	<p>For using KE gain = PE loss</p> <p>AEF</p>
(ii)	<p><math>0.4g \sin \theta - R = 0.4v^2/0.8</math>  <math>[0.4g \sin \theta - R = 4.64 \theta - 3.2 \sin \theta]</math>  <math>R = 7.12 \sin \theta - 4.64 \theta</math></p>	<p>M1                  A1                  M1                  A1                  [4]</p>	<p>For applying Newton's second law to P and                  using <math>a = v^2/r</math></p> <p>For substituting for <math>v^2</math></p> <p>AG</p>
(iii)	<p><math>R(1.53) = 0.01(48\dots)</math>, <math>R(1.54) = -0.02(9\dots)</math> or                  simply <math>R(1.53) &gt; 0</math> and <math>R(1.54) &lt; 0</math></p> <p><math>R(1.53) \times R(1.54) &lt; 0 \rightarrow 1.53 &lt; \alpha &lt; 1.54</math></p>	<p>M1                  A1                    M1                  A1                  [4]</p>	<p>For substituting 1.53 and 1.54 into <math>R(\theta)</math></p> <p>For using the idea that if <math>R(1.53)</math> and  <math>R(1.54)</math> are of opposite signs then R is zero                  (and thus P leaves the surface) for some                  value of <math>\theta</math> between 1.53 and 1.54.</p> <p>AG</p>
7(i)	<p><math>T_{AP} = 19.6e/1.6</math> and <math>T_{BP} = 19.6(1.6-e)/1.6</math></p> <p><math>0.5g \sin 30^\circ + 12.25(1.6 - e) = 12.25e</math>                  Distance AP is 2.5m</p>	<p>M1                  A1                  M1                  A1ft                  A1                  [5]</p>	<p>For using <math>T = \lambda e/L</math></p> <p>For resolving forces parallel to the plane</p>
(ii)	<p>Extensions of AP and BP are <math>0.9 + x</math> and  <math>0.7 - x</math> respectively</p> <p><math>0.5g \sin 30^\circ + 19.6(0.7 - x)/1.6</math>  <math>- 19.6(0.9 + x)/1.6 = 0.5 \ddot{x}</math></p> <p><math>\ddot{x} = -49x</math></p> <p>Period is 0.898 s</p>	<p>B1                    B1ft                  B1                  M1                  A1                  [5]</p>	<p>AG</p> <p>For stating <math>k &lt; 0</math> and using <math>T = 2\pi/\sqrt{-k}</math></p>
(iii)	<p><math>2.8^2 = 49(0.5^2 - x^2)</math>  <math>x^2 = 0.09</math></p> <p><math>x = 0.3</math> and <math>-0.3</math></p>	<p>M1                  A1ft                  A1                    A1ft                  [4]</p>	<p>For using <math>v^2 = \omega^2(A^2 - x^2)</math> where <math>\omega^2 = -k</math>                  ft incorrect value of k                  May be implied by a value of x                  ft incorrect value of k or incorrect value of  <math>x^2</math> (stated)</p>