OCR
RECOGNISING ACHIEVEMENT

**ADVANCED GCE**

# MATHEMATICS

Further Pure Mathematics 3

**4727**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**
None

**Friday 29 January 2010**
**Morning**

**Duration:** 1 hour 30 minutes

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**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.



**Turn over**

1 Determine whether the lines

$$\frac{x-1}{1}=\frac{y+2}{-1}=\frac{z+4}{2} \qquad \text{and} \qquad \frac{x+3}{2}=\frac{y-1}{3}=\frac{z-5}{4}$$

intersect or are skew. **[5]**

2 $H$ denotes the set of numbers of the form $a+b\sqrt{5}$, where $a$ and $b$ are rational. The numbers are combined under multiplication.

**(i)** Show that the product of any two members of $H$ is a member of $H$. **[2]**

It is now given that, for $a$ and $b$ not both zero, $H$ forms a group under multiplication.

**(ii)** State the identity element of the group. **[1]**

**(iii)** Find the inverse of $a+b\sqrt{5}$. **[2]**

**(iv)** With reference to your answer to part **(iii)**, state a property of the number 5 which ensures that every number in the group has an inverse. **[1]**

3 Use the integrating factor method to find the solution of the differential equation

$$\frac{dy}{dx}+2y=e^{-3x}$$

for which $y=1$ when $x=0$. Express your answer in the form $y=f(x)$. **[6]**

4 **(i)** Write down, in cartesian form, the roots of the equation $z^4=16$. **[2]**

**(ii)** Hence solve the equation $w^4=16(1-w)^4$, giving your answers in cartesian form. **[5]**

5 A regular tetrahedron has vertices at the points

$$A\left(0,\,0,\,\tfrac{2}{3}\sqrt{6}\right),\quad B\left(\tfrac{2}{3}\sqrt{3},\,0,\,0\right),\quad C\left(-\tfrac{1}{3}\sqrt{3},\,1,\,0\right),\quad D\left(-\tfrac{1}{3}\sqrt{3},\,-1,\,0\right).$$

**(i)** Obtain the equation of the face $ABC$ in the form

$$x+\sqrt{3}y+\left(\tfrac{1}{2}\sqrt{2}\right)z=\tfrac{2}{3}\sqrt{3}.$$ **[5]**

(Answers which only verify the given equation will not receive full credit.)

**(ii)** Give a geometrical reason why the equation of the face $ABD$ can be expressed as

$$x-\sqrt{3}y+\left(\tfrac{1}{2}\sqrt{2}\right)z=\tfrac{2}{3}\sqrt{3}.$$ **[2]**

**(iii)** Hence find the cosine of the angle between two faces of the tetrahedron. **[4]**

**6** The variables $x$ and $y$ satisfy the differential equation

$$\frac{d^2y}{dx^2} + 16y = 8\cos 4x.$$

**(i)** Find the complementary function of the differential equation. **[2]**

**(ii)** Given that there is a particular integral of the form $y = px\sin 4x$, where $p$ is a constant, find the general solution of the equation. **[6]**

**(iii)** Find the solution of the equation for which $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. **[4]**

**7** **(i)** Solve the equation $\cos 6\theta = 0$, for $0 < \theta < \pi$. **[3]**

**(ii)** By using de Moivre's theorem, show that

$$\cos 6\theta \equiv (2\cos^2\theta - 1)(16\cos^4\theta - 16\cos^2\theta + 1).$$ **[5]**

**(iii)** Hence find the exact value of

$$\cos\left(\tfrac{1}{12}\pi\right)\cos\left(\tfrac{5}{12}\pi\right)\cos\left(\tfrac{7}{12}\pi\right)\cos\left(\tfrac{11}{12}\pi\right),$$

justifying your answer. **[5]**

**8** The function f is defined by $\mathrm{f} : x \mapsto \frac{1}{2-2x}$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$. The function g is defined by $\mathrm{g}(x) = \mathrm{ff}(x)$.

**(i)** Show that $\mathrm{g}(x) = \frac{1-x}{1-2x}$ and that $\mathrm{gg}(x) = x$. **[4]**

It is given that f and g are elements of a group $K$ under the operation of composition of functions. The element e is the identity, where $\mathrm{e} : x \mapsto x$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$.

**(ii)** State the orders of the elements f and g. **[2]**

**(iii)** The inverse of the element f is denoted by h. Find $\mathrm{h}(x)$. **[2]**

**(iv)** Construct the operation table for the elements e, f, g, h of the group $K$. **[4]**