



**ADVANCED GCE**  
**MATHEMATICS**  
Further Pure Mathematics 3

**4727**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Friday 29 January 2010**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## 2

- 1 Determine whether the lines

$$\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z+4}{2} \quad \text{and} \quad \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-5}{4}$$

intersect or are skew.

[5]

- 2  $H$  denotes the set of numbers of the form  $a + b\sqrt{5}$ , where  $a$  and  $b$  are rational. The numbers are combined under multiplication.

- (i) Show that the product of any two members of  $H$  is a member of  $H$ . [2]

It is now given that, for  $a$  and  $b$  not both zero,  $H$  forms a group under multiplication.

- (ii) State the identity element of the group. [1]

- (iii) Find the inverse of  $a + b\sqrt{5}$ . [2]

- (iv) With reference to your answer to part (iii), state a property of the number 5 which ensures that every number in the group has an inverse. [1]

- 3 Use the integrating factor method to find the solution of the differential equation

$$\frac{dy}{dx} + 2y = e^{-3x}$$

for which  $y = 1$  when  $x = 0$ . Express your answer in the form  $y = f(x)$ .

[6]

- 4 (i) Write down, in cartesian form, the roots of the equation  $z^4 = 16$ . [2]

- (ii) Hence solve the equation  $w^4 = 16(1-w)^4$ , giving your answers in cartesian form. [5]

- 5 A regular tetrahedron has vertices at the points

$$A(0, 0, \frac{2}{3}\sqrt{6}), \quad B(\frac{2}{3}\sqrt{3}, 0, 0), \quad C(-\frac{1}{3}\sqrt{3}, 1, 0), \quad D(-\frac{1}{3}\sqrt{3}, -1, 0).$$

- (i) Obtain the equation of the face  $ABC$  in the form

$$x + \sqrt{3}y + (\frac{1}{2}\sqrt{2})z = \frac{2}{3}\sqrt{3}. \quad [5]$$

(Answers which only verify the given equation will not receive full credit.)

- (ii) Give a geometrical reason why the equation of the face  $ABD$  can be expressed as

$$x - \sqrt{3}y + (\frac{1}{2}\sqrt{2})z = \frac{2}{3}\sqrt{3}. \quad [2]$$

- (iii) Hence find the cosine of the angle between two faces of the tetrahedron. [4]

3

6 The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{d^2y}{dx^2} + 16y = 8 \cos 4x.$$

(i) Find the complementary function of the differential equation. [2]

(ii) Given that there is a particular integral of the form  $y = px \sin 4x$ , where  $p$  is a constant, find the general solution of the equation. [6]

(iii) Find the solution of the equation for which  $y = 2$  and  $\frac{dy}{dx} = 0$  when  $x = 0$ . [4]

7 (i) Solve the equation  $\cos 6\theta = 0$ , for  $0 < \theta < \pi$ . [3]

(ii) By using de Moivre's theorem, show that

$$\cos 6\theta \equiv (2 \cos^2 \theta - 1)(16 \cos^4 \theta - 16 \cos^2 \theta + 1). \quad [5]$$

(iii) Hence find the exact value of

$$\cos\left(\frac{1}{12}\pi\right) \cos\left(\frac{5}{12}\pi\right) \cos\left(\frac{7}{12}\pi\right) \cos\left(\frac{11}{12}\pi\right),$$

justifying your answer. [5]

8 The function  $f$  is defined by  $f : x \mapsto \frac{1}{2 - 2x}$  for  $x \in \mathbb{R}$ ,  $x \neq 0$ ,  $x \neq \frac{1}{2}$ ,  $x \neq 1$ . The function  $g$  is defined by  $g(x) = ff(x)$ .

(i) Show that  $g(x) = \frac{1 - x}{1 - 2x}$  and that  $gg(x) = x$ . [4]

It is given that  $f$  and  $g$  are elements of a group  $K$  under the operation of composition of functions. The element  $e$  is the identity, where  $e : x \mapsto x$  for  $x \in \mathbb{R}$ ,  $x \neq 0$ ,  $x \neq \frac{1}{2}$ ,  $x \neq 1$ .

(ii) State the orders of the elements  $f$  and  $g$ . [2]

(iii) The inverse of the element  $f$  is denoted by  $h$ . Find  $h(x)$ . [2]

(iv) Construct the operation table for the elements  $e, f, g, h$  of the group  $K$ . [4]

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