



General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics

MFP1

Unit Further Pure 1

Wednesday 13 January 2010 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed).

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The quadratic equation

$$3x^2 - 6x + 1 = 0$$

has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)

(b) Show that $\alpha^3 + \beta^3 = 6$. (3 marks)

(c) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. (4 marks)

2 The complex number z is defined by

$$z = 1 + i$$

(a) Find the value of z^2 , giving your answer in its simplest form. (2 marks)

(b) Hence show that $z^8 = 16$. (2 marks)

(c) Show that $(z^*)^2 = -z^2$. (2 marks)

3 Find the general solution of the equation

$$\sin\left(4x + \frac{\pi}{4}\right) = 1 \quad (4 \text{ marks})$$

4 It is given that

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$

and that \mathbf{I} is the 2×2 identity matrix.

(a) Show that $(\mathbf{A} - \mathbf{I})^2 = k\mathbf{I}$ for some integer k . (3 marks)

(b) Given further that

$$\mathbf{B} = \begin{bmatrix} 1 & 3 \\ p & 1 \end{bmatrix}$$

find the integer p such that

$$(\mathbf{A} - \mathbf{B})^2 = (\mathbf{A} - \mathbf{I})^2 \quad (4 \text{ marks})$$

5 (a) Explain why $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}} dx$ is an improper integral. (1 mark)

(b) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}} dx$; (3 marks)

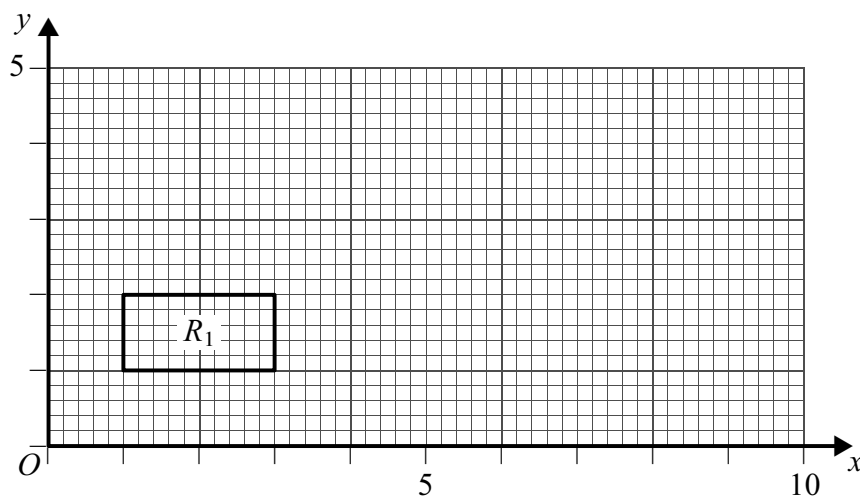
(ii) $\int_0^{\frac{1}{16}} x^{-\frac{5}{4}} dx$. (3 marks)

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6 [Figure 1, printed on the insert, is provided for use in this question.]

The diagram shows a rectangle R_1 .



- (a) The rectangle R_1 is mapped onto a second rectangle, R_2 , by a transformation with matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.
- (i) Calculate the coordinates of the vertices of the rectangle R_2 . (2 marks)
- (ii) On **Figure 1**, draw the rectangle R_2 . (1 mark)
- (b) The rectangle R_2 is rotated through 90° clockwise about the origin to give a third rectangle, R_3 .
- (i) On **Figure 1**, draw the rectangle R_3 . (2 marks)
- (ii) Write down the matrix of the rotation which maps R_2 onto R_3 . (1 mark)
- (c) Find the matrix of the transformation which maps R_1 onto R_3 . (2 marks)

7 A curve C has equation $y = \frac{1}{(x-2)^2}$.

- (a) (i) Write down the equations of the asymptotes of the curve C . (2 marks)
- (ii) Sketch the curve C . (2 marks)
- (b) The line $y = x - 3$ intersects the curve C at a point which has x -coordinate α .
- (i) Show that α lies within the interval $3 < x < 4$. (2 marks)
- (ii) Starting from the interval $3 < x < 4$, use interval bisection twice to obtain an interval of width 0.25 within which α must lie. (3 marks)

8 (a) Show that

$$\sum_{r=1}^n r^3 + \sum_{r=1}^n r$$

can be expressed in the form

$$kn(n+1)(an^2 + bn + c)$$

where k is a rational number and a , b and c are integers. (4 marks)

(b) Show that there is exactly one positive integer n for which

$$\sum_{r=1}^n r^3 + \sum_{r=1}^n r = 8 \sum_{r=1}^n r^2 \quad (5 \text{ marks})$$

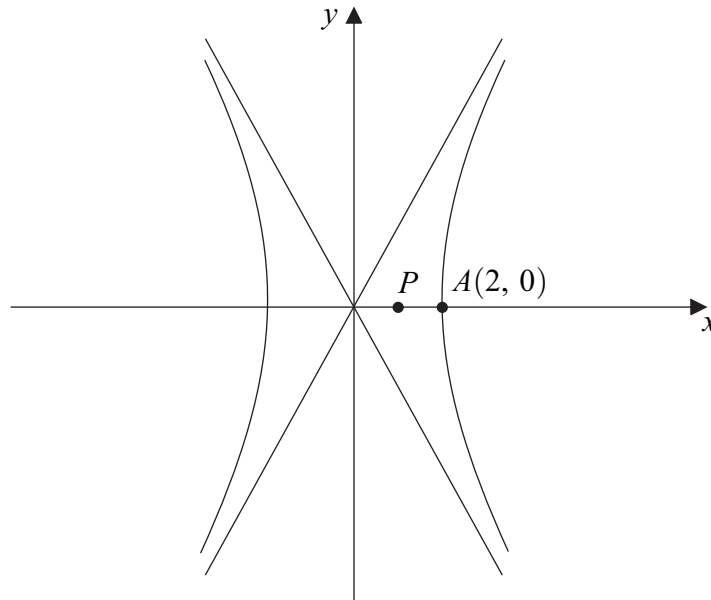
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9 The diagram shows the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and its asymptotes.



The constants a and b are positive integers.

The point A on the hyperbola has coordinates $(2, 0)$.

The equations of the asymptotes are $y = 2x$ and $y = -2x$.

(a) Show that $a = 2$ and $b = 4$. (4 marks)

(b) The point P has coordinates $(1, 0)$. A straight line passes through P and has gradient m . Show that, if this line intersects the hyperbola, the x -coordinates of the points of intersection satisfy the equation

$$(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0 \quad (4 \text{ marks})$$

(c) Show that this equation has equal roots if $3m^2 = 16$. (3 marks)

(d) There are two tangents to the hyperbola which pass through P . Find the coordinates of the points at which these tangents touch the hyperbola.

(No credit will be given for solutions based on differentiation.) (5 marks)

END OF QUESTIONS

Figure 1 (for use in Question 6)

