



General Certificate of Education
Advanced Level Examination
January 2011

Mathematics

MFP2

Unit Further Pure 2

Wednesday 19 January 2011 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

- 1 (a)** Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 4 + 3i| = 5 \quad (3 \text{ marks})$$

- (b) (i)** Indicate on your diagram the point P representing z_1 , where both

$$|z_1 - 4 + 3i| = 5 \quad \text{and} \quad \arg z_1 = 0 \quad (1 \text{ mark})$$

- (ii)** Find the value of $|z_1|$. (1 mark)
-

- 2 (a)** Given that

$$u_r = \frac{1}{6}r(r+1)(4r+11)$$

show that

$$u_r - u_{r-1} = r(2r+3) \quad (3 \text{ marks})$$

- (b)** Hence find the sum of the first hundred terms of the series

$$1 \times 5 + 2 \times 7 + 3 \times 9 + \dots + r(2r+3) + \dots \quad (3 \text{ marks})$$

- 3 (a)** Show that $(1+i)^3 = 2i - 2$. (2 marks)

- (b)** The cubic equation

$$z^3 - (5+i)z^2 + (9+4i)z + k(1+i) = 0$$

where k is a real constant, has roots α , β and γ .

It is given that $\alpha = 1 + i$.

- (i)** Find the value of k . (3 marks)

- (ii)** Show that $\beta + \gamma = 4$. (1 mark)

- (iii)** Find the values of β and γ . (5 marks)
-

4 (a) Prove that the curve

$$y = 12 \cosh x - 8 \sinh x - x$$

has exactly one stationary point. (7 marks)

(b) Given that the coordinates of this stationary point are (a, b) , show that $a + b = 9$.
(4 marks)

5 (a) Given that $u = \sqrt{1 - x^2}$, find $\frac{du}{dx}$. (2 marks)

(b) Use integration by parts to show that

$$\int_0^{\frac{\sqrt{3}}{2}} \sin^{-1} x \, dx = a\sqrt{3}\pi + b$$

where a and b are rational numbers. (6 marks)

6 (a) Given that

$$x = \ln(\sec t + \tan t) - \sin t$$

show that

$$\frac{dx}{dt} = \sin t \tan t \quad (4 \text{ marks})$$

(b) A curve is given parametrically by the equations

$$x = \ln(\sec t + \tan t) - \sin t, \quad y = \cos t$$

The length of the arc of the curve between the points where $t = 0$ and $t = \frac{\pi}{3}$ is denoted by s .

Show that $s = \ln p$, where p is an integer. (6 marks)

7 (a) Given that

$$f(k) = 12^k + 2 \times 5^{k-1}$$

show that

$$f(k+1) - 5f(k) = a \times 12^k$$

where a is an integer.

(3 marks)

(b) Prove by induction that $12^n + 2 \times 5^{n-1}$ is divisible by 7 for all integers $n \geq 1$.

(4 marks)

8 (a) Express in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$:

(i) $4(1 + i\sqrt{3})$;

(ii) $4(1 - i\sqrt{3})$.

(3 marks)

(b) The complex number z satisfies the equation

$$(z^3 - 4)^2 = -48$$

Show that $z^3 = 4 \pm 4\sqrt{3}i$.

(2 marks)

(c) (i) Solve the equation

$$(z^3 - 4)^2 = -48$$

giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

(5 marks)

(ii) Illustrate the roots on an Argand diagram.

(3 marks)

(d) (i) Explain why the sum of the roots of the equation

$$(z^3 - 4)^2 = -48$$

is zero.

(1 mark)

(ii) Deduce that $\cos \frac{\pi}{9} + \cos \frac{3\pi}{9} + \cos \frac{5\pi}{9} + \cos \frac{7\pi}{9} = \frac{1}{2}$.

(3 marks)