

General Certificate of Education
January 2008
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 2

MPC2

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

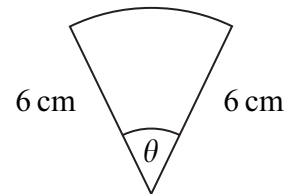
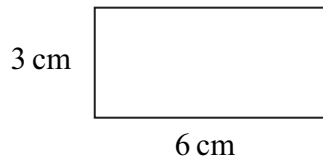
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The diagrams show a rectangle of length 6 cm and width 3 cm, and a sector of a circle of radius 6 cm and angle θ radians.



The area of the rectangle is twice the area of the sector.

- (a) Show that $\theta = 0.5$. (3 marks)
- (b) Find the perimeter of the sector. (3 marks)

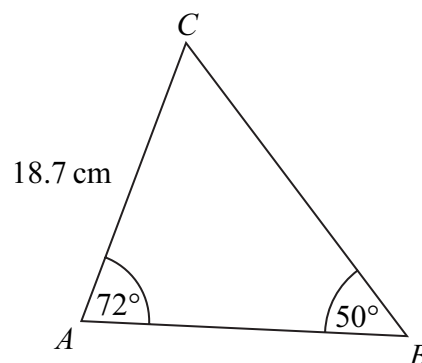
- 2 The arithmetic series

$$51 + 58 + 65 + 72 + \dots + 1444$$

has 200 terms.

- (a) Write down the common difference of the series. (1 mark)
- (b) Find the 101st term of the series. (2 marks)
- (c) Find the sum of **the last** 100 terms of the series. (2 marks)

- 3 The diagram shows a triangle ABC . The length of AC is 18.7 cm, and the sizes of angles BAC and ABC are 72° and 50° respectively.



- (a) Show that the length of $BC = 23.2$ cm, correct to the nearest 0.1 cm. (3 marks)
- (b) Calculate the area of triangle ABC , giving your answer to the nearest cm^2 . (3 marks)

- 4 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

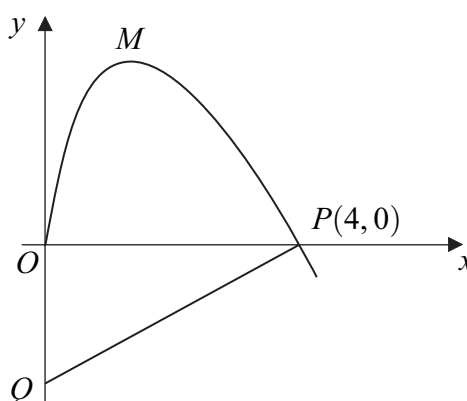
$$\int_0^3 \sqrt{x^2 + 3} \, dx$$

giving your answer to three decimal places.

(4 marks)

- 5 A curve, drawn from the origin O , crosses the x -axis at the point $P(4, 0)$.

The normal to the curve at P meets the y -axis at the point Q , as shown in the diagram.



The curve, defined for $x \geq 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

- (a) (i) Find $\frac{dy}{dx}$. (3 marks)
- (ii) Show that the gradient of the curve at $P(4, 0)$ is -2 . (2 marks)
- (iii) Find an equation of the normal to the curve at $P(4, 0)$. (3 marks)
- (iv) Find the y -coordinate of Q and hence find the area of triangle OPQ . (3 marks)
- (v) The curve has a maximum point M . Find the x -coordinate of M . (3 marks)
- (b) (i) Find $\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$. (3 marks)
- (ii) Find the total area of the region bounded by the curve and the lines PQ and QO . (3 marks)

Turn over ►

6 (a) Using the binomial expansion, or otherwise:

(i) express $(1 + x)^3$ in ascending powers of x ; (2 marks)

(ii) express $(1 + x)^4$ in ascending powers of x . (2 marks)

(b) Hence, or otherwise:

(i) express $(1 + 4x)^3$ in ascending powers of x ; (2 marks)

(ii) express $(1 + 3x)^4$ in ascending powers of x . (2 marks)

(c) Show that the expansion of

$$(1 + 3x)^4 - (1 + 4x)^3$$

can be written in the form

$$px^2 + qx^3 + rx^4$$

where p , q and r are integers.

(2 marks)

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x .

(1 mark)

(b) Given that

$$\log_a y = 2 \log_a 3 + \log_a 4 + 1$$

express y in terms of a , giving your answer in a form **not** involving logarithms.

(3 marks)

8 (a) Sketch the graph of $y = 3^x$, stating the coordinates of the point where the graph crosses the y -axis. (2 marks)

(b) Describe a single geometrical transformation that maps the graph of $y = 3^x$:

(i) onto the graph of $y = 3^{2x}$; (2 marks)

(ii) onto the graph of $y = 3^{x+1}$. (2 marks)

(c) (i) Using the substitution $Y = 3^x$, show that the equation

$$9^x - 3^{x+1} + 2 = 0$$

can be written as

$$(Y - 1)(Y - 2) = 0 \quad (2 \text{ marks})$$

(ii) Hence show that the equation $9^x - 3^{x+1} + 2 = 0$ has a solution $x = 0$ and, by using logarithms, find the other solution, giving your answer to four decimal places. (4 marks)

9 (a) Given that

$$\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$$

show that

$$\cos \theta = -\frac{1}{2} \quad (4 \text{ marks})$$

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3 \cos 3x$$

giving all solutions in degrees in the interval $0^\circ < x < 180^\circ$. (4 marks)

END OF QUESTIONS

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