Version 1.0

# General Certificate of Education
# June 2010

## Mathematics MM1B

## Mechanics 1B

***Mark Scheme***

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.



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**Key to mark scheme and abbreviations used in marking**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| –*x* EE | deduct *x* marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

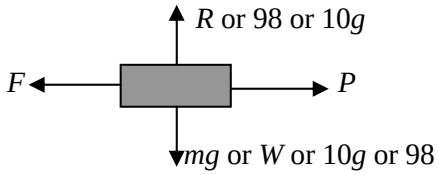
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MM1B**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | 30 seconds | B1 | 1 | B1: Correct statement of time. |
| **(b)** | $s_1 = \frac{1}{2}\times 40\times 20 = 400$ m | M1<br>A1 | 2 | M1: A method for calculating the first distance. Must see 40 and $\frac{1}{2}$.<br>A1: Correct distance. |
| | **OR** | | | |
| | $s_1 = \frac{1}{2}\times(20+0)\times 40 = 400\,\text{m}$ | (M1)<br>(A1) | | |
| | **OR** | | | |
| | $a = -\frac{20}{40} = -\frac{1}{2}$ | | | Note on third method: Must see $-\frac{1}{2}$ or $-\frac{20}{40}$ plus attempt to find distance for M1. |
| | $0^2 = 20^2 + 2\left(-\frac{1}{2}\right)s$ | (M1) | | |
| | $s = 20^2 = 400$ m | (A1) | | |
| **(c)** | $s_2 = \frac{1}{2}\times 50\times 20 = 500\,\text{m}$ | M1 | | M1: Method for finding the second distance and calculating the total distance. |
| | **OR** | | | |
| | $s_2 = \frac{1}{2}\times(0+20)\times 50 = 500\,\text{m}$ | (M1) | | |
| | **OR** | | | |
| | $a = \frac{20}{50} = \frac{2}{5}$ | | | |
| | $20^2 = 0^2 + 2\left(\frac{2}{5}\right)s$ | (M1) | | Note on third method: Must see $\frac{2}{5}$ or $\frac{20}{50}$ plus attempt to find distance. |
| | $s = 20^2 \times \frac{5}{4} = 500$ m | | | |
| | Total = 400 + 500 = 900 m | A1F | 2 | A1F: Correct total distance. Award the follow through mark for correct addition of 500 and their answer to (b). |
| **(d)** | $v_{AVERAGE} = \frac{900}{120} = 7.5$ ms$^{-1}$ | M1<br>A1F | 2 | M1: Their total distance divided by 120<br>A1F: Correct average speed based on their answer to (c). |
| **(e)** | $120\times 20 - 900 = 1500$ m | M1A1F | 2 | M1: Multiplication of 20 and 120 to find distance.<br>Note: Award M1 if 2400 seen in this part.<br>A1F: Correct difference based on their answer to (c) provided final answer is positive. |
| | **Total** | | **9** | |

**MM1B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **2(a)** |  | B1 | 1 | B1: Correct force diagram with arrows and labels.<br>Note: Award mark if forces drawn on the diagram in the question.<br>Note: Do not accept 10 kg for the weight.<br>Note: Do not accept $\mu R$ or $0.5R$ for $F$. |
| **(b)(i)** | $(R = 10\times 9.8 =) 98$ N | B1 | 1 | B1: Correct normal reaction. Accept 10$g$. No need to see the letter $R$ or working. |
| **(ii)** | $(F \leq) 0.5\times 98$<br>$(F \leq) 49$ | B1F | 1 | B1: Correct maximum value for friction. Accept 5$g$.<br>No need to see the letter $F$ or any working. Ignore any inequalities.<br>For FT, must be 0.5 of candidate's answer to (b)(i). |
| **(iii)** | $(F =) 30$ N | B1 | 1 | B1: Correct friction. Allow – 30. |
| **(c)** | $80 - 49 = 10\,a$ | M1A1F | | M1: Three term equation motion, containing 80, candidate's 49 and 10$a$ (not 10$ga$) in any combination.<br>A1F: Correct equation including signs. |
| | $a = 3.1\,\text{ms}^{-2}$ | A1F | 3 | A1F: Correct acceleration.<br>FT candidate's answer to (b)(ii). |
| | **Total** | | **7** | |
| | | | | Allow use of $g$ = 9.81<br><br>(b)(i) 98. 1 B1<br>(b)(ii) 49.05 or 49.1 or 49 B1<br>(c) 3.095 or 3.09 or 3.1 M1A1A1 |

**MM1B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | $6\begin{bmatrix}2\\4\end{bmatrix}+m\begin{bmatrix}3\\-2\end{bmatrix}=6\begin{bmatrix}1\\3\end{bmatrix}+m\begin{bmatrix}7\\b\end{bmatrix}$ | M1<br>A1 | | M1: Four term conservation of momentum equation. Allow sign errors.<br>A1: Correct equation with correct signs. Vector equation may be implied by later correct working in this part of the question. |
| | $6\times2+3m=6\times1+7m$ | A1 | | A1: Correct equation for correct component. |
| | $12+3m=6+7m$<br>$6=4m$ | | | |
| | $m=1.5$ | A1 | 4 | A1: Correct $m$.<br><br>Example if only $12+3m=6-7m$ without a vector equation award M1A0A0A0. |
| **(b)** | $6\times4+1.5\times(-2)=6\times3+1.5b$ | B1F | | B1F: Correct equation using $m$ or candidates $m$ from (a).<br>B1F: Correct $b$ from candidate's $m$ from (a). |
| | $24-3=18+1.5b$<br>$3=1.5b$ | | | |
| | $b=2$ | B1F | 2 | Note: $b=\frac{6}{m}-2$ . |
| | **Total** | | **6** | |
| | | | | Consistent use of $mg$ instead of $m$ throughout penalise 1 mark. |

**MM1B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** | $50\cos\theta = 60\cos 48°$ | M1A1 | | M1: Equation for two forces, with both forces resolved horizontally in the same way. (Accept $50\sin\theta = 60\sin 48°$ for M1.)<br>A1: Correct equation. |
| | **OR** | | | |
| | $50\cos\theta = 60\sin 42°$ | (M1)<br>(A1) | | |
| | **OR** (from vector triangle and sine rule) | | | |
| | $\dfrac{50}{\sin 42°} = \dfrac{60}{\sin(90-\theta)}$ | (M1)<br>(A1) | | (M1: Use of sine rule with 50, 60 and 42°.)<br>(A1: Correct equation.) |
| | **OR** (from Lami's Theorem) | | | |
| | $\dfrac{50}{\sin 138°} = \dfrac{60}{\sin(90+\theta)}$ | (M1)<br>(A1) | | (M1: Use of Lami's Theorem with 50, 60 and 138°.)<br>(A1: Correct equation.) |
| | For example:<br>$\theta = \cos^{-1}\left(\dfrac{60\cos 48°}{50}\right)$ | dM1 | | dM1: Solving for $\theta$.<br>A1: Correct $\theta$. |
| | $= 36.59°$<br>$= 36.6°$ (to 3SF) | A1 | 4 | Note: Final answer of 63.1° from using resolving incorrectly with sines award M1A0dM1A0.<br>Accept 36.5 (truncation) and 36.7 and AWRT 36.6. |
| **(b)** | $50\sin 36.59° + 60\sin 48° = 9.8\,m$ | M1<br>A1F | | M1: Three term vertical equation, including *mg* with forces resolved vertically in the same way (accept $50\cos 36.59° + 60\cos 48° = 9.8\,m$ for M1).<br>A1F: Correct equation. |
| | **OR** correct equivalent, for example: | | | |
| | $50\sin 36.59° + 60\cos 42° = 9.8\,m$ | (M1)<br>(A1F) | | |
| | **OR** (from vector triangle and sine rule) | | | |
| | $\dfrac{50}{\sin 42°} = \dfrac{mg}{\sin 84.6°}$ | (M1)<br>(A1F) | | (M1: Use of vector triangle and sine rule.) |
| | **OR** (from Lami's Theorem) | | | |
| | $\dfrac{50}{\sin 138°} = \dfrac{60}{\sin 95.4°}$ | (M1)<br>(A1F) | | (M1: Use of Lami's Theorem.) |
| | For example:<br>$m = \dfrac{50\sin 36.59° + 60\sin 48°}{9.8} = 7.59$ | A1 | 3 | A1: Correct value for *m* CAO.<br>Accept 7.58, AWRT 7.6. |
| | **Total** | | **7** | |
| | | | | Allow use of $g = 9.81$<br>(b) 7.58 M1A1A1 |

**MM1B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** | $(v=)\sqrt{30^2+100^2}$<br>$=104.4$<br>$=104\ \text{ms}^{-1}\ \ (\text{to 3SF})$ | M1A1<br><br>A1 | 3 | M1: Equation or expression to find $v$ based on Pythagoras. Must be +.<br>For example: 10 900 oe scores M1.<br>A1: Correct equation or expression, with square root.<br>A1: Correct $v$. Accept 104.4. |
| **(b)** | $\theta=\tan^{-1}\left(\frac{30}{100}\right)$ or $\tan^{-1}\left(\frac{100}{30}\right)$<br>$=017°$<br>**OR**<br>$\theta=\sin^{-1}\left(\frac{30}{104.4}\right)$ or $\sin^{-1}\left(\frac{100}{104.4}\right)$<br>$=017°$<br>**OR**<br>$\theta=\cos^{-1}\left(\frac{100}{104.4}\right)$ or $\cos^{-1}\left(\frac{30}{104.4}\right)$<br>$=017°$ | M1<br><br>A1F<br><br>(M1)<br><br>(A1F)<br><br>(M1)<br><br>(A1F) | 2 | M1: Trigonometric equation to find $\alpha$.<br><br>A1F: Correct $\alpha$. Follow through incorrect answer from (b).<br>Note: Subtracting 17 etc from other values such as 360 or 90 can not be ignored and will score M1.<br>Accept 16 or 17 or 16.6 or 16.7 or 16.8. Also accept all of these with a zero in front, eg 016. |
| | **Total** | | **5** | |

**MM1B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)** | $12g - T = 12a$ | M1A1 | | M1: Three term equation of motion, with $12g$ (or 117.6), $12a$ (not $12ga$) and $T$.<br>A1: Correct equation |
| | $T - 8g = 8a$ | M1A1 | | M1: Three term equation of motion, with $8g$ (or 78.4), $8a$ (not $8ga$) and $T$.<br>A1: Correct equation |
| | $4g = 20a$ | | | |
| | $a\left(=\frac{4g}{20}\right) = 1.96 \text{ ms}^{-2}$ AG | A1 | 5 | A1: Correct acceleration from correct working.<br><br>Note: Do not penalise candidates who consistently use signs in the opposite direction throughout, provided they give their final answer as 1.96. If final answer is – 1.96 don't award final A1 mark.<br><br>Special Case:<br>Whole String Method $4g = 20a$ and<br>$a = \frac{4g}{20} = 1.96$ OE M1A1A1 |
| **(b)** | $T = 8g + 8\times 1.96 = 94.1\text{N}$ | M1A1 | 2 | M1: Use of three term equation of motion to find $T$, with $a$ = 1.96.<br>A1: Correct tension.<br>Accept 94.08. |
| **(c)(i)** | $v = 0 + 1.96\times 2 = 3.92 \text{ ms}^{-1}$ | M1A1 | 2 | M1: Use of constant acceleration equation to find $v$, with $a$ = 1.96 and $u$ = 0.<br>A1: Correct $v$.<br>Using $s$ = 4 scores M0. |
| **(ii)** | $v^2 = 3.92^2 + 2\times 9.8\times 4$ | M1<br>A1F | | M1: Use of constant acceleration equation to find $v$, with $a = \pm 9.8$ and $u \neq 0$.<br>A1F: Correct equation. FT initial velocity from (c)(i). |
| | $v = 9.68 \text{ ms}^{-1}$ | A1F | 3 | A1F: Correct $v$. FT initial velocity from (c)(i).<br>For example 11.8 from 7.84. |

**MM1B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **(c)(iii)** | $4 = \frac{1}{2}(-3.92 + 9.68)t$ | M1A1<br>A1 | | M1: Use of $s = \frac{1}{2}(u+v)t$<br>A1: Correct values.<br>A1: Correct signs. |
| | $t = 1.39$ | dM1<br>A1 | 5 | dM1: Solving for *t*.<br>A1: Correct *t*. |
| | **OR** | | | |
| | $-4 = 3.92t - 4.9t^2$<br>$4.9t^2 - 3.92t - 4 = 0$<br>$t = \frac{3.92 \pm \sqrt{3.92^2 - 4\times 4.9\times(-4)}}{2\times 4.9}$ | (M1)<br>(A1)<br>(A1) | | M1: Forming a quadratic with candidates *u* from (c)(i) or *v* from (c)(ii)with 4.9 or 9.8.<br>A1: Correct terms in quadratic.<br>A1: Correct signs in quadratic. |
| | $t = 1.39$ or $t = -0.588$<br>$t = 1.39$ | (dM1)<br>(A1) | | dM1: Solving quadratic (do not penalise for negative discriminant).<br>A1: Correct root seen (other root does not need to be seen). |
| | **OR** | | | |
| | $t_{up} + t_{down} = 0.4 + 0.4 + 0.588$<br>$= 1.39$ (to 3SF) | (M1)<br>(A1)<br>(dM1)<br>(A1)<br>(A1) | | M1: Finding total time from two or three times.<br>A1: 0.4 or 0.8 seen.<br>dM1: Finding second or third time for downward motion.<br>A1: Obtaining 0.588 or 0.988.<br>A1: 1.39. Accept 1.38. |
| | **OR** | | | |
| | $9.68 = -3.92 + 9.8t$<br>$t = \frac{13.6}{9.8} = 1.39$ | (M1)<br>(A1)<br>(A1)<br>(dM1)<br>(A1) | | M1: Use of $v = u + at$<br>A1: Correct values.<br>A1: Correct signs.<br>dM1: Solving for *t*<br>A1: Correct *t* |
| | **Total** | | **17** | |
| | | | | Use of $g = 9.81$<br><br>(a) 1.962 M1A1M1A1A0<br>(b) 94.2 M1A1<br>(c) (ii) 9.69 M1A1A1<br>(c) (iii) 1.39 M1A1A1dM1A1 |

**MM1B(cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)** | $10\,\mathbf{a} = 9\,\mathbf{i} + 12\,\mathbf{j}$ | M1 | | M1: Application of Newton's second Law with $m = 10$ in vector form. |
| | $\mathbf{a} = (0.9\,\mathbf{i} + 1.2\,\mathbf{j})\ \text{ms}^{-2}$ | A1 | 2 | A1: Correct acceleration. If acceleration incorrect follow their value through for the rest of this question. |
| **(b)(i)** | $\mathbf{r}(5) =$ | | | |
| | $(2.2\,\mathbf{i} + 1\,\mathbf{j}) \times 5 + \frac{1}{2}(0.9\,\mathbf{i} + 1.2\,\mathbf{j}) \times 5^2$ | M1 | | M1: Use of constant acceleration to find position vector at $t = 5$, with $\mathbf{u} \neq 0\,\mathbf{i} + 0\,\mathbf{j}$. |
| | $= 22.25\,\mathbf{i} + 20\,\mathbf{j}$ | A1F | | A1F: Correct position vector, for candidate's acceleration which must be a vector. Allow 22.3**i** + 20**j**. |
| | $d = \sqrt{22.25^2 + 20^2} = 29.9$ metres | dM1<br>A1F | 4 | dM1: Calculation of distance from position vector. Must see + sign.<br>A1F: Correct distance, for their acceleration.<br>Accept 30 from 22.3**i** + 20**j**. |
| **(ii)** | $\mathbf{v} = (2.2\,\mathbf{i} + 1\,\mathbf{j}) + (0.9\,\mathbf{i} + 1.2\,\mathbf{j})t$ | M1<br>A1F | 2 | M1: Use of constant acceleration equation to find an expression for **v,** with $\mathbf{u} \neq 0\,\mathbf{i} + 0\,\mathbf{j}$.<br>A1F: Correct **v** for their acceleration. |
| **(iii)** | $\mathbf{v} = (2.2 + 0.9t)\,\mathbf{i} + (1 + 1.2t)\,\mathbf{j}$ | M1 | | M1: Equation involving both **i** and **j** components of their velocity. Could have incorrect signs, for example $2.2 + 0.9t = -(1 + 1.2t)$. |
| | $2.2 + 0.9t = 1 + 1.2t$ | A1F | | A1F: Correct equation. |
| | $1.2 = 0.3t$<br>$t = 4$ | A1F | 3 | A1F: Correct time, for their acceleration. |
| | **Total** | | **11** | |

**MM1B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** | $14.7\sin\alpha - 9.8t\,(=0)$ | M1A1 | | M1: Equation for vertical velocity being zero at highest point. Must have $\sin\alpha$ with $\pm 9.8$. A1: Correct equation. |
| | $t = \dfrac{14.7\sin\alpha}{9.8} = \dfrac{3\sin\alpha}{2}$ **AG** | A1 | 3 | A1: Correct result from correct working. |
| | **OR** | | | |
| | $14.7\sin\alpha T - 4.9T^2\ (=0)$ | | | |
| | $T = \dfrac{14.7\sin\alpha}{4.9} = 3\sin\alpha$ | (M1) | | All marks awarded for last line, from correct working. |
| | $t = \dfrac{3\sin\alpha}{2}$ | (A1) (A1) | | |
| **(b)(i)** | $7 = 14.7\sin\alpha\left(\dfrac{3\sin\alpha}{2}\right) - 4.9\left(\dfrac{3\sin\alpha}{2}\right)^2$ | M1 A1 | | M1: Expression including vertical displacement at height 7, using expression from part (a) and with $\pm g$ or equivalent. A1: Correct expression. |
| | $7 = 11.025\sin^2\alpha$ | dM1 | | dM1: Simplified expression with $\sin^2\alpha$. |
| | $\alpha = \sin^{-1}\left(\sqrt{\dfrac{7}{11.025}}\right) = 52.8°$ | dM1 A1 | 5 | dM1: Finding an angle. Must have previous dM1 mark. A1: Correct angle. Accept 52.7°, 52.9°. |
| | **OR** | | | |
| | $0^2 = (14.7\sin\alpha)^2 + 2\times(-9.8)\times 7$ | (M1) | | |
| | $\sin^2\alpha = \dfrac{2\times 9.8\times 7}{14.7^2}$ | (A1) (dM1) | | |
| | $\alpha = 52.8°$ | (dM1) (A1) | | |
| **(ii)** | $OA = 14.7\cos 52.8°\times 3\sin 52.8°$ | B1M1 | | B1: Use of $3\sin\alpha$ with their $\alpha$. M1: Finding horizontal displacement. including $14.7\cos\alpha$ with $3\sin\alpha$ or $\dfrac{3\sin\alpha}{2}$. |
| | $OA = 21.2$ m | A1 | 3 | A1: Correct distance. Accept 21.3 m. |
| **(c)** | Ball is a particle/No spin.<br>No air resistance/No wind/Constant acceleration of 9.8/Only force is weight. | B1<br>B1 | 2 | B1: Particle assumption.<br>B1: Air resistance assumption. |
| | **Total** | | **13** | |
| | **TOTAL** | | **75** | |
| | | | | Use of $g$ = 9.81:<br>(a) M1A1A0<br>(b)(i) 52.8° or 52.9° M1A1dM1dM1A1<br>(b)(ii) 21.2 B1M1A1 |