

General Certificate of Education Advanced Level Examination June 2013

# **Mathematics**

# MM2B

## Unit Mechanics 2B

## Thursday 13 June 2013 9.00 am to 10.30 am

#### For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

#### Time allowed

• 1 hour 30 minutes

#### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

#### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

#### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



2

1 A particle, of mass 3 kg, moves along a straight line. At time *t* seconds, the displacement, *s* metres, of the particle from the origin is given by

$$s = 8t^3 + 15$$

- (a) Find the velocity of the particle at time t. (2 marks)
- (b) Find the magnitude of the resultant force acting on the particle when t = 2. (4 marks)
- 2 Carol, a circus performer, is on a swing. She jumps off the swing and lands in a safety net. When Carol leaves the swing, she has a speed of  $7 \text{ m s}^{-1}$  and she is at a height of 8 metres above the safety net.

Carol is to be modelled as a particle of mass 52 kg being acted upon only by gravity.

- (a) Find the kinetic energy of Carol when she leaves the swing. (2 marks)
- (b) Show that the kinetic energy of Carol when she hits the net is 5350 J, correct to three significant figures. (3 marks)
- (c) Find the speed of Carol when she hits the net. (3 marks)
- 3 A particle, of mass 10 kg, moves on a smooth horizontal plane. At time *t* seconds, the acceleration of the particle is given by

$$\{(40t+3t^2)\mathbf{i}+20e^{-4t}\mathbf{j}\}\,\mathrm{m\,s}^{-2}$$

where the vectors **i** and **j** are perpendicular unit vectors.

- (a) At time t = 1, the velocity of the particle is  $(6\mathbf{i} 5e^{-4}\mathbf{j}) \,\mathrm{m}\,\mathrm{s}^{-1}$ . Find the velocity of the particle at time t. (5 marks)
- (b) Calculate the initial speed of the particle.
- 4 A uniform plank AB, of length 6 m, has mass 25 kg. It is supported in equilibrium in a horizontal position by two vertical inextensible ropes. One of the ropes is attached to the plank at the point P and the other rope is attached to the plank at the point Q, where AP = 1 m and QB = 0.8 m, as shown in the diagram.



- (a) (i) Find the tension in each rope.
  - (ii) State how you have used the fact that the plank is uniform in your solution. (1 mark)



(5 marks)

(3 marks)

3

(b) A particle of mass  $m \log B$  is attached to the plank at point B, and the tension in each rope is now the same.

Find *m*.

(6 marks)

5 Tom is travelling on a train which is moving at a constant speed of  $15 \,\mathrm{m\,s^{-1}}$  on a horizontal track. Tom has placed his mobile phone on a rough horizontal table. The coefficient of friction between the phone and the table is 0.2.

The train moves round a bend of constant radius. The phone does not slide as the train travels round the bend.

Model the phone as a particle moving round part of a circle, with centre O and radius r metres.

Find the least possible value of r.

(4 marks)

(6 marks)

6 A car accelerates from rest along a straight horizontal road.

The car's engine produces a constant horizontal force of magnitude 4000 N.

At time t seconds, the speed of the car is  $v m s^{-1}$ , and a resistance force of magnitude 40v newtons acts upon the car.

The mass of the car is 1600 kg.

(a) Show that 
$$\frac{dv}{dt} = \frac{100 - v}{40}$$
. (2 marks)

- (b) Find the velocity of the car at time t.
- 7 A train, of mass 22 tonnes, moves along a straight horizontal track. A constant resistance force of 5000 N acts on the train. The power output of the engine of the train is 240 kW.

Find the acceleration of the train when its speed is  $20 \text{ m s}^{-1}$ . (6 marks)



4

8 A bead, of mass m, moves on a smooth circular ring, of radius a and centre O, which is fixed in a vertical plane. At P, the highest point on the ring, the speed of the bead is 2u; at Q, the lowest point on the ring, the speed of the bead is 5u.

(a) Show that 
$$u = \sqrt{\frac{4ag}{21}}$$
. (4 marks)

(b) S is a point on the ring so that angle POS is  $60^{\circ}$ , as shown in the diagram.



Find, in terms of m and g, the magnitude of the reaction of the ring on the bead when the bead is at S. (5 marks)

**9** Two particles, *A* and *B*, are connected by a light elastic string that passes through a hole at a point *O* in a rough horizontal table. The edges of the hole are smooth. Particle *A* has a mass of 8 kg and particle *B* has a mass of 3 kg.

The elastic string has natural length 3 metres and modulus of elasticity 60 newtons.

Initially, particle A is held 3.5 metres from the point O on the surface of the table and particle B is held at a point 2 metres vertically below O.

The coefficient of friction between the table and particle A is 0.4.

The two particles are released from rest.

- (a) (i) Show that initially particle A moves towards the hole in the table. (3 marks)
  - (ii) Show that initially particle *B* also moves towards the hole in the table. (2 marks)
- (b) Calculate the initial elastic potential energy in the string. (2 marks)
- (c) Particle A comes permanently to rest when it has moved 0.46 metres, at which time particle B is still moving upwards.

Calculate the distance that particle B has moved when it is at rest for the first time. (7 marks)

