

2. The continuous random variable Y has cumulative distribution function

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{4}(y^3 - 4y^2 + ky) & 0 \leq y \leq 2 \\ 1 & y > 2 \end{cases}$$

where k is a constant.

(a) Find the value of k . (2)

(b) Find the probability density function of Y , specifying it for all values of y . (3)

(c) Find $P(Y > 1)$. (2)

4. The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(3 + 2x - x^2) & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$ (3)

(b) Find the mode of X . (2)

(c) Use algebraic integration to find $E(X)$. (4)

By comparing your answers to parts (b) and (c),

(d) describe the skewness of X , giving a reason for your answer. (2)

5. In a village shop the customers must join a queue to pay. The number of customers joining the queue in a 10 minute interval is modelled by a Poisson distribution with mean 3

Find the probability that

(a) exactly 4 customers join the queue in the next 10 minutes, (2)

(b) more than 10 customers join the queue in the next 20 minutes. (3)

When a customer reaches the front of the queue the customer pays the assistant. The time each customer takes paying the assistant, T minutes, has a continuous uniform distribution over the interval $[0, 5]$. The random variable T is independent of the number of people joining the queue.

(c) Find $P(T > 3.5)$ (1)

In a random sample of 5 customers, the random variable C represents the number of customers who took more than 3.5 minutes paying the assistant.

(d) Find $P(C \geq 3)$ (3)

Bethan has just reached the front of the queue and starts paying the assistant.

(e) Find the probability that in the next 4 minutes Bethan finishes paying the assistant and no other customers join the queue. (4)



- 6. Frugal bakery claims that their packs of 10 muffins contain on average 80 raisins per pack. A Poisson distribution is used to describe the number of raisins per muffin.

A muffin is selected at random to test whether or not the mean number of raisins per muffin has changed.

- (a) Find the critical region for a two-tailed test using a 10% level of significance. The probability of rejection in each tail should be less than 0.05 (4)

- (b) Find the actual significance level of this test. (2)

The bakery has a special promotion claiming that their muffins now contain even more raisins.

A random sample of 10 muffins is selected and is found to contain a total of 95 raisins.

- (c) Use a suitable approximation to test the bakery's claim. You should state your hypotheses clearly and use a 5% level of significance. (8)



7. As part of a selection procedure for a company, applicants have to answer all 20 questions of a multiple choice test. If an applicant chooses answers at random the probability of choosing a correct answer is 0.2 and the number of correct answers is represented by the random variable X .

(a) Suggest a suitable distribution for X . (2)

Each applicant gains 4 points for each correct answer but loses 1 point for each incorrect answer. The random variable S represents the final score, in points, for an applicant who chooses answers to this test at random.

(b) Show that $S = 5X - 20$ (2)

(c) Find $E(S)$ and $\text{Var}(S)$. (4)

An applicant who achieves a score of at least 20 points is invited to take part in the final stage of the selection process.

(d) Find $P(S \geq 20)$ (4)

Cameron is taking the final stage of the selection process which is a multiple choice test consisting of 100 questions. He has been preparing for this test and believes that his chance of answering each question correctly is 0.4

(e) Using a suitable approximation, estimate the probability that Cameron answers more than half of the questions correctly. (5)



