



General Certificate of Education
Advanced Level Examination
January 2013

Mathematics

MFP2

Unit Further Pure 2

Wednesday 23 January 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

2

1 (a) Show that

$$12 \cosh x - 4 \sinh x = 4e^x + 8e^{-x} \quad (2 \text{ marks})$$

(b) Solve the equation

$$12 \cosh x - 4 \sinh x = 33$$

giving your answers in the form $k \ln 2$. (5 marks)2 Two loci, L_1 and L_2 , in an Argand diagram are given by

$$L_1 : |z + 6 - 5i| = 4\sqrt{2}$$

$$L_2 : \arg(z + i) = \frac{3\pi}{4}$$

The point P represents the complex number $-2 + i$.(a) Verify that the point P is a point of intersection of L_1 and L_2 . (2 marks)(b) Sketch L_1 and L_2 on one Argand diagram. (6 marks)(c) The point Q is also a point of intersection of L_1 and L_2 . Find the complex number that is represented by Q . (2 marks)3 (a) Show that $\frac{1}{5r-2} - \frac{1}{5r+3} = \frac{A}{(5r-2)(5r+3)}$, stating the value of the constant A . (2 marks)

(b) Hence use the method of differences to show that

$$\sum_{r=1}^n \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)} \quad (4 \text{ marks})$$

(c) Find the value of

$$\sum_{r=1}^{\infty} \frac{1}{(5r-2)(5r+3)} \quad (1 \text{ mark})$$



4 The roots of the equation

$$z^3 - 5z^2 + kz - 4 = 0$$

are α , β and γ .

(a) (i) Write down the value of $\alpha + \beta + \gamma$ and the value of $\alpha\beta\gamma$. (2 marks)

(ii) Hence find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. (2 marks)

(b) The value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ is -4 .

(i) Explain why α , β and γ cannot all be real. (1 mark)

(ii) By considering $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, find the possible values of k . (4 marks)

5 (a) Using the definition $\tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}}$, show that, for $|x| < 1$,

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (3 \text{ marks})$$

(b) Hence, or otherwise, show that $\frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2}$. (3 marks)

(c) Use integration by parts to show that

$$\int_0^{\frac{1}{2}} 4 \tanh^{-1} x \, dx = \ln \left(\frac{3^m}{2^n} \right)$$

where m and n are positive integers. (5 marks)

6 A curve is defined parametrically by

$$x = t^3 + 5, \quad y = 6t^2 - 1$$

The arc length between the points where $t = 0$ and $t = 3$ on the curve is s .

(a) Show that $s = \int_0^3 3t\sqrt{t^2 + A} \, dt$, stating the value of the constant A . (4 marks)

(b) Hence show that $s = 61$. (4 marks)

Turn over ►



- 7 The polynomial $p(n)$ is given by $p(n) = (n - 1)^3 + n^3 + (n + 1)^3$.
- (a) (i) Show that $p(k + 1) - p(k)$, where k is a positive integer, is a multiple of 9. (3 marks)
- (ii) Prove by induction that $p(n)$ is a multiple of 9 for all integers $n \geq 1$. (4 marks)
- (b) Using the result from part (a)(ii), show that $n(n^2 + 2)$ is a multiple of 3 for any positive integer n . (2 marks)
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- 8 (a) Express $-4 + 4\sqrt{3}i$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. (3 marks)
- (b) (i) Solve the equation $z^3 = -4 + 4\sqrt{3}i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. (4 marks)
- (ii) The roots of the equation $z^3 = -4 + 4\sqrt{3}i$ are represented by the points P , Q and R on an Argand diagram.
- Find the area of the triangle PQR , giving your answer in the form $k\sqrt{3}$, where k is an integer. (3 marks)
- (c) By considering the roots of the equation $z^3 = -4 + 4\sqrt{3}i$, show that

$$\cos \frac{2\pi}{9} + \cos \frac{4\pi}{9} + \cos \frac{8\pi}{9} = 0 \quad (4 \text{ marks})$$

