General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Wednesday 20 June 2007 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the **blue** AQA booklet of formulae and statistical tables • an insert for use in Questions 5 and 9 (enclosed). You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(b) The matrix **M** represents a combination of an enlargement of scale factor p and a reflection in a line L. State the value of p and write down the equation of L. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and **I** is the 2×2 identity matrix. *(2 marks)*

2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. *(3 marks)*

(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. *(4 marks)*

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$z - 3\mathrm{i}z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$z - 3\mathrm{i}z^* = 16$$ *(3 marks)*

4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{1}{4}$. *(2 marks)*

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta}$$ *(3 marks)*

5 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = ab^x$$

where a and b are constants.

The following approximate values of x and y have been found.

x	1	2	3	4
y	3.84	6.14	9.82	15.7

(a) Complete the table in **Figure 1**, showing values of x and Y, where $Y = \log_{10} y$. Give each value of Y to three decimal places. *(2 marks)*

(b) Show that, if $y = ab^x$, then x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

(c) Draw on **Figure 2** a linear graph relating x and Y. *(2 marks)*

(d) Hence find estimates for the values of a and b. *(4 marks)*

6 Find the general solution of the equation

$$\sin\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(6 marks)*

7 A curve has equation

$$y = \frac{3x - 1}{x + 2}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes. *(5 marks)*

(c) Hence, or otherwise, solve the inequality

$$0 < \frac{3x - 1}{x + 2} < 3$$ *(2 marks)*

8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\displaystyle\int_0^1 \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right) \mathrm{d}x$; *(4 marks)*

(b) $\displaystyle\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x} \mathrm{d}x$. *(4 marks)*

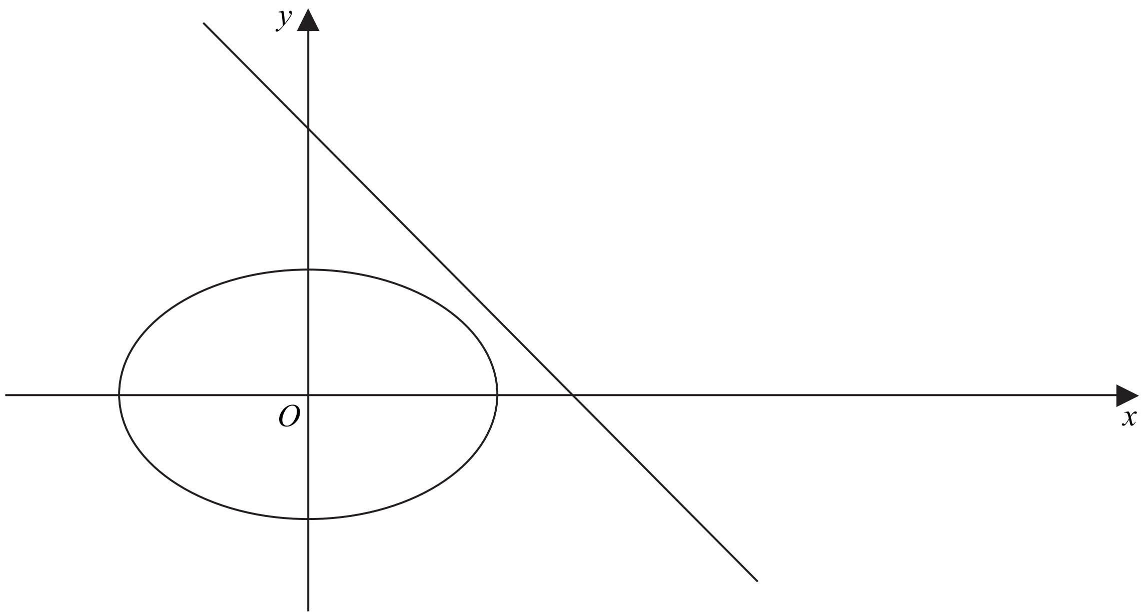
9 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



(a) Write down the exact coordinates of the points where the curve with equation $\frac{x^2}{2} + y^2 = 1$ intersects the coordinate axes. *(2 marks)*

(b) The curve is translated k units in the positive x direction, where k is a constant. Write down, in terms of k, the equation of the curve after this translation. *(2 marks)*

(c) Show that, if the line $x + y = 2$ intersects the **translated** curve, the x-coordinates of the points of intersection must satisfy the equation

$$3x^2 - 2(k+4)x + (k^2 + 6) = 0$$ *(4 marks)*

(d) Hence find the two values of k for which the line $x + y = 2$ is a tangent to the translated curve. Give your answer in the form $p \pm \sqrt{q}$, where p and q are integers. *(4 marks)*

(e) On **Figure 3**, show the translated curves corresponding to these two values of k. *(3 marks)*

END OF QUESTIONS

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Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

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Insert

Insert for use in **Questions 5 and 9**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

P93925/Jun07/MFP1 6/6/6/

Figure 1 (for use in Question 5)

x	1	2	3	4
Y	0.584			

Figure 2 (for use in Question 5)

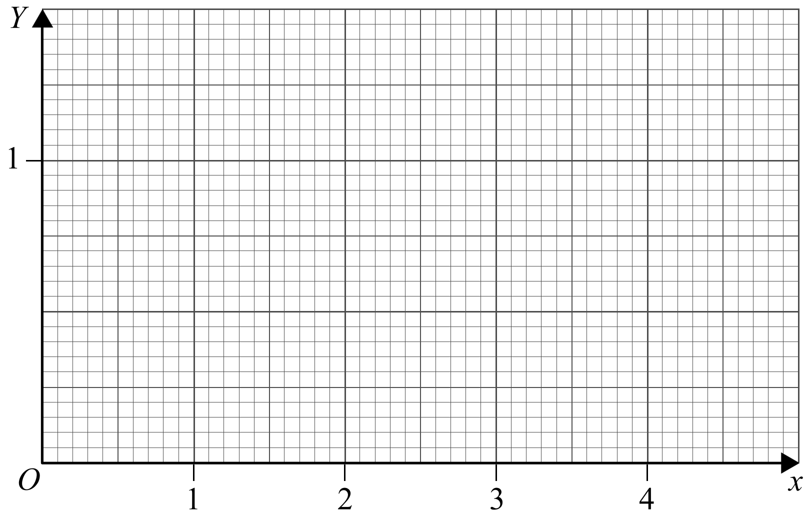

Figure 3 (for use in Question 9)