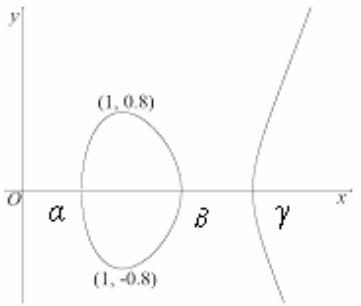
4726 Further Pure Mathematics 2

1	**(i)**	Give $1 + 2x + (2x)^2/2$	M1	Reasonable 3 term attempt e.g. allow $2x^2/2$
		Get $1 + 2x + 2x^2$	A1	cao
				SC Reasonable attempt at f′(0) and f″(0) M1
				Get $1+2x+2x^2$ cao A1
	(ii)	$\ln((1+2x+2x^2) + (1-2x+2x^2))$ =	M1	Attempt to sub for e^{2x} and e^{-2x}
		$\ln(2+4x^2)$ =	A1√	On their part **(i)**
		$\ln 2 + \ln(1 + 2x^2)$	M1	Use of log law in reasonable expression
		$\ln 2 + 2x^2$	A1	cao
				SC Use of Maclaurin for f′(x) and f″(x) M1
				One correct A1
				Attempt f(0), f′(0) and f″(0) M1
				Get cao A1
2	**(i)**	x_2 = 1.8913115	B1	x_2 correct; allow answers which round
		x_3 = 1.8915831	B1√	For any other from their working
		x_4 = 1.8915746	B1	For all three correct
	(ii)	e_3/e_2 = - 0.031(1)	M1	Subtraction and division on their values; allow ±
		e_4/e_3 = -0.036(5)	A1	Or answers which round to −0.031 and −0.037
		State f′(α) ≈ $e_3/e_2 \approx e_4/e_3$	B1√	Using their values but only if approx. equal; allow differentiation if correct conclusion; allow gradient for f′
3	**(i)**	Diff. sin $y = x$	M1	Implicit diff. to $dy/dx = \pm(1/\cos y)$
		Use $\sin^2 + \cos^2 = 1$ to A.G.	A1	Clearly derived; ignore ±
		Justify +	B1	e.g graph/ principal values
	(ii)	Get $2/(\sqrt{(1-4x^2)} + 1/(\sqrt{(1-y^2)}\, dy/dx = 0$	M1	Attempt implicit diff. and chain rule; allow e.g. $(1-2x^2)$ or $a/\sqrt{(1-4x^2)}$
			A1	
		Find $y = \sqrt{3}/2$	M1	Method leading to y
		Get $-2\sqrt{3}/3$	A1√	AEEF; from their a above
				SC Write $\sin(\frac{1}{2}\pi - \sin^{-1}2x) = \cos(\sin^{-1}2x)$ B1
				Attempt to diff. as above M1
				Replace x in reasonable dy/dx and attempt to tidy M1
				Get result above A1

4	**(i)**	Let $x=\cosh\theta$ such that $dx = \sinh\theta\, d\theta$	M1	
		Clearly use $\cosh^2 - \sinh^2 = 1$	A1	Clearly derive A.G.
	(ii)	Replace $\cosh^2\theta$	M1	Allow $a(\cosh 2\theta \pm 1)$
		Attempt to integrate their expression	M1	Allow $b\sinh 2\theta \pm a\theta$
		Get $\frac{1}{4}\sinh 2\theta + \frac{1}{2}\theta\ (+c)$	A1	
		Clearly replace for x to A.G.	B1	Condone no $+c$
				SC Use expo. def^n; three terms M1
				Attempt to integrate M1
				Get $\frac{1}{8}(e^{2\theta}-e^{-2\theta}) + \frac{1}{2}\theta\ (+c)$ A1
				Clearly replace for x to A.G. B1
5	**(i)**	(**a**) State ($x=$) α	B1	
		None of roots	B1	No explanation needed
		(**b**) Impossible to say	B1	
		All roots can be derived	B1	Some discussion of values close to 1 or 2 or central leading to correct conclusion
	(ii)		B1	Correct x for y=0; allow 0.591, 1.59, 2.31
			B1	Turning at (1,0.8) and/or (1,-0.8)
			B1	Meets x-axis at 90^0
			B1	Symmetry in x-axis; allow
6	**(i)**	Correct definitions used	B1	
		Attempt at $(e^x-e^{-x})^2/4 + 1$	M1	Allow $(e^x+e^{-x})^2+1$; allow /2
		Clearly derive A.G.	A1	
	(ii)	Form a quadratic in $\sinh x$	M1	
		Attempt to solve	M1	Factors or formula
		Get $\sinh x = -\frac{1}{2}$ or 3	A1	
		Use correct ln expression	M1	On their answer(s) seen once
		Get $\ln(-\frac{1}{2}+\frac{\sqrt{5}}{2})$ and $\ln(3+\sqrt{10})$	A1	
7	**(i)**	$OP = 3 + 2\cos\alpha$		
		$OQ = 3 + 2\cos(\frac{1}{2}\pi+\alpha)$ $= 3 - 2\sin\alpha$	M1	Any other unsimplified value
		Similarly $OR = 3 - 2\cos\alpha$	M1	Attempt at simplification of at least two correct expressions
		$OS = 3 + 2\sin\alpha$		
		Sum = 12	A1	cao
	(ii)	Correct formula with attempt at r^2	M1	Need not be expanded, but three terms if it is
		Square r correctly	A1	
		Attempt to replace $\cos^2\theta$ with $a(\cos 2\theta \pm 1)$	M1	
		Integrate their expression	A1√	Need three terms
		Get $\frac{11\pi}{4} - 1$	A1	cao

8	**(i)**	Area = $\int 1/(x+1)\,dx$	B1	Include or imply correct limits
		Use limits to $\ln(n+1)$	B1	
		Compare area under curve to areas of rectangles	B1	Justify inequality
		Sum of areas = $1x(\frac{1}{2} + \frac{1}{3} + \ldots + 1/(n+1))$	M1	Sum seen or implied as 1 x y values
		Clear detail to A.G.	A1	Explanation required e.g. area of last rectangle at $x=n$, area under curve to $x=n$
	(ii)	Show or explain areas of rectangles above curve	M1	
		Areas of rectangles (as above) > area under curve	A1	First and last heights seen or implied; A.G.
	(iii)	Add 1 to both sides in **(i)** to make $\sum(1/r)$	B1	Must be clear addition
		Add $1/(n+1)$ to both sides in **(ii)** to make $\sum(1/r)$	B1	Must be clear addition; A.G.
	(iv)	State divergent	B1	Allow not convergent
		Explain e.g. $\ln(n+1)\to\infty$ as $n\to\infty$	B1	
9	**(i)**	Require denom. = 0	B1	
		Explain why denom. $\neq 0$	B1	Attempt to solve, explain always > 0 etc.
	(ii)	Set up quadratic in x	M1	
		Get $2yx^2-4x+(2a^2y+3a) =0$	A1	
		Use $b^2 \geq 4ac$ for real x	M1	Produce quadratic inequality in y from their quad.; allow use of = or <
		Attempt to solve their inequality	M1	Factors or formula
		Get $y > 1/2a$ and $y < -2/a$	A1	Justified from graph
				SC Attempt diff. by quot./product rule M1
				Solve $dy/dx =0$ for two values of x M1
				Get $x=2a$ and $x=-a/2$ A1
				Attempt to find two y values M1
				Get correct inequalities (graph used to justify them) A1
	(iii)	Split into two separate integrals	M1	
		Get $k\ln(x^2+a^2)$	A1	Or $p\ln(2x^2+2a^2)$
		Get $k_1\tan^{-1}(x/a)$	A1	k_1 not involving a
		Use limits and attempt to simplify	M1	
		Get $\ln 2.5 - 1.5\tan^{-1}2 + 3\pi/8$	A1	AEEF
				SC Sub. $x = a\tan\theta$ and $dx = a\sec^2\theta\,d\theta$ M1
				Reduce to $\int p\tan\theta - p_1\,d\theta$ (ignore limits here) A1
				Integrate to $p\ln(\sec\theta) - p_1\theta$ A1
				Use limits (old or new) and attempt to simplify M1
				Get answer above A1