General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS MPC1
Unit Pure Core 1

ASSESSMENT and QUALIFICATIONS ALLIANCE

Wednesday 10 January 2007   1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

MPC1

Answer **all** questions.

1 The polynomial p(x) is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of p(x), show that $k = 10$. *(2 marks)*

  (ii) Express p(x) as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when p(x) is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the *x*-axis and the *y*-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

2 The line *AB* has equation $3x + 5y = 8$ and the point *A* has coordinates $(6, -2)$.

(a) (i) Find the gradient of *AB*. *(2 marks)*

  (ii) Hence find an equation of the straight line which is perpendicular to *AB* and which passes through *A*. *(3 marks)*

(b) The line *AB* intersects the line with equation $2x + 3y = 3$ at the point *B*. Find the coordinates of *B*. *(3 marks)*

(c) The point *C* has coordinates $(2, k)$ and the distance from *A* to *C* is 5. Find the **two** possible values of the constant *k*. *(3 marks)*

3 (a) Express $\dfrac{\sqrt{5} + 3}{\sqrt{5} - 2}$ in the form $p\sqrt{5} + q$, where p and q are integers. *(4 marks)*

(b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. *(1 mark)*

  (ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form. *(3 marks)*

4 A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2 \qquad \text{(3 marks)}$$

(b) Write down:

   (i) the coordinates of C; *(1 mark)*

   (ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the x-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q.

   (i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 3x - 10 = 0 \qquad \text{(3 marks)}$$

   (ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q. *(2 marks)*

   (iii) Hence find the coordinates of the midpoint of PQ. *(2 marks)*

Turn over for the next question

Turn over ▶

5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.
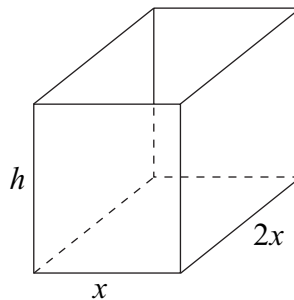

The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, $V\ \text{m}^3$, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3}$$

(1 mark)

(b) (i) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\dfrac{\mathrm{d}^2 V}{\mathrm{d}x^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.
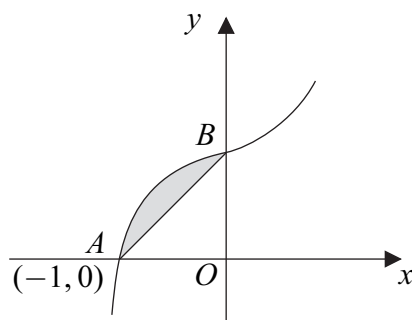
The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

(ii) Find $\int (3x^5 + 2x + 5)\ \mathrm{d}x$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

7 The quadratic equation $(k + 1)x^2 + 12x + (k - 4) = 0$ has real roots.

(a) Show that $k^2 - 3k - 40 \leqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

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