

General Certificate of Education
June 2009
Advanced Level Examination



MATHEMATICS
Unit Further Pure 2

MFP2

Friday 5 June 2009 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Given that $z = 2e^{\frac{\pi i}{12}}$ satisfies the equation

$$z^4 = a(1 + \sqrt{3}i)$$

where a is real:

- (a) find the value of a ; (3 marks)
- (b) find the other three roots of this equation, giving your answers in the form $re^{i\theta}$,
where $r > 0$ and $-\pi < \theta \leq \pi$. (5 marks)

2 (a) Given that

$$\frac{1}{4r^2 - 1} = \frac{A}{2r - 1} + \frac{B}{2r + 1}$$

find the values of A and B . (2 marks)

(b) Use the method of differences to show that

$$\sum_{r=1}^n \frac{1}{4r^2 - 1} = \frac{n}{2n + 1} \quad (3 \text{ marks})$$

(c) Find the least value of n for which $\sum_{r=1}^n \frac{1}{4r^2 - 1}$ differs from 0.5 by less than 0.001.
(3 marks)

3 The cubic equation

$$z^3 + pz^2 + 25z + q = 0$$

where p and q are real, has a root $\alpha = 2 - 3i$.

- (a) Write down another non-real root, β , of this equation. (1 mark)
- (b) Find:
- (i) the value of $\alpha\beta$; (1 mark)
- (ii) the third root, γ , of the equation; (3 marks)
- (iii) the values of p and q . (3 marks)

4 (a) Sketch the graph of $y = \tanh x$. (2 marks)

(b) Given that $u = \tanh x$, use the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x} to show that

$$x = \frac{1}{2} \ln \left(\frac{1+u}{1-u} \right) \quad (6 \text{ marks})$$

(c) (i) Show that the equation

$$3 \operatorname{sech}^2 x + 7 \tanh x = 5$$

can be written as

$$3 \tanh^2 x - 7 \tanh x + 2 = 0 \quad (2 \text{ marks})$$

(ii) Show that the equation

$$3 \tanh^2 x - 7 \tanh x + 2 = 0$$

has only one solution for x .

Find this solution in the form $\frac{1}{2} \ln a$, where a is an integer. (5 marks)

5 (a) Prove by induction that, if n is a positive integer,

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \quad (5 \text{ marks})$$

(b) Hence, given that

$$z = \cos \theta + i \sin \theta$$

show that

$$z^n + \frac{1}{z^n} = 2 \cos n\theta \quad (3 \text{ marks})$$

(c) Given further that $z + \frac{1}{z} = \sqrt{2}$, find the value of

$$z^{10} + \frac{1}{z^{10}} \quad (4 \text{ marks})$$

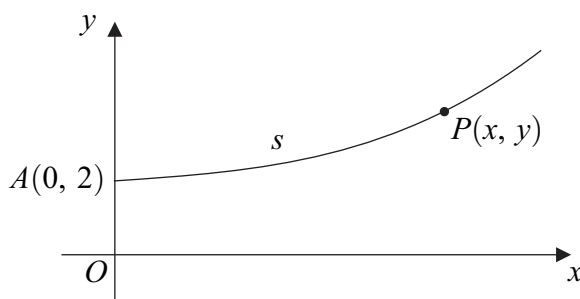
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Turn over ►

- 6 (a) Two points, A and B , on an Argand diagram are represented by the complex numbers $2 + 3i$ and $-4 - 5i$ respectively. Given that the points A and B are at the ends of a diameter of a circle C_1 , express the equation of C_1 in the form $|z - z_0| = k$. (4 marks)
- (b) A second circle, C_2 , is represented on the Argand diagram by the equation $|z - 5 + 4i| = 4$. Sketch on one Argand diagram both C_1 and C_2 . (3 marks)
- (c) The points representing the complex numbers z_1 and z_2 lie on C_1 and C_2 respectively and are such that $|z_1 - z_2|$ has its maximum value. Find this maximum value, giving your answer in the form $a + b\sqrt{5}$. (5 marks)
- 7 The diagram shows a curve which starts from the point A with coordinates $(0, 2)$. The curve is such that, at every point P on the curve,

$$\frac{dy}{dx} = \frac{1}{2}s$$

where s is the length of the arc AP .



- (a) (i) Show that

$$\frac{ds}{dx} = \frac{1}{2}\sqrt{4 + s^2} \quad (3 \text{ marks})$$

- (ii) Hence show that

$$s = 2 \sinh \frac{x}{2} \quad (4 \text{ marks})$$

- (iii) Hence find the cartesian equation of the curve. (3 marks)

- (b) Show that

$$y^2 = 4 + s^2 \quad (2 \text{ marks})$$

END OF QUESTIONS