



**ADVANCED GCE  
MATHEMATICS**

Further Pure Mathematics 3

**4727**

Candidates answer on the answer booklet.

**OCR supplied materials:**

- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Monday 13 June 2011  
Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

2

1 A line  $l$  has equation  $\frac{x-1}{5} = \frac{y-6}{6} = \frac{z+3}{-7}$  and a plane  $p$  has equation  $x + 2y - z = 40$ .

(i) Find the acute angle between  $l$  and  $p$ . [4]

(ii) Find the perpendicular distance from the point  $(1, 6, -3)$  to  $p$ . [2]

2 It is given that  $z = e^{i\theta}$ , where  $0 < \theta < 2\pi$ , and  $w = \frac{1+z}{1-z}$ .

(i) Prove that  $w = i \cot \frac{1}{2}\theta$ . [3]

(ii) Sketch separate Argand diagrams to show the locus of  $z$  and the locus of  $w$ . You should show the direction in which each locus is described when  $\theta$  increases in the interval  $0 < \theta < 2\pi$ . [3]

3 The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} + 4y = 5 \cos 3x.$$

(i) Find the complementary function. [2]

(ii) Hence, or otherwise, find the general solution. [7]

(iii) Find the approximate range of values of  $y$  when  $x$  is large and positive. [2]

4 A group  $G$ , of order 8, is generated by the elements  $a, b, c$ .  $G$  has the properties

$$a^2 = b^2 = c^2 = e, \quad ab = ba, \quad bc = cb, \quad ca = ac,$$

where  $e$  is the identity.

(i) Using these properties and basic group properties as necessary, prove that  $abc = cba$ . [2]

The operation table for  $G$  is shown below.

	$e$	$a$	$b$	$c$	$bc$	$ca$	$ab$	$abc$
$e$	$e$	$a$	$b$	$c$	$bc$	$ca$	$ab$	$abc$
$a$	$a$	$e$	$ab$	$ca$	$abc$	$c$	$b$	$bc$
$b$	$b$	$ab$	$e$	$bc$	$c$	$abc$	$a$	$ca$
$c$	$c$	$ca$	$bc$	$e$	$b$	$a$	$abc$	$ab$
$bc$	$bc$	$abc$	$c$	$b$	$e$	$ab$	$ca$	$a$
$ca$	$ca$	$c$	$abc$	$a$	$ab$	$e$	$bc$	$b$
$ab$	$ab$	$b$	$a$	$abc$	$ca$	$bc$	$e$	$c$
$abc$	$abc$	$bc$	$ca$	$ab$	$a$	$b$	$c$	$e$

(ii) List all the subgroups of order 2. [2]

(iii) List five subgroups of order 4. [3]

(iv) Determine whether all the subgroups of  $G$  which are of order 4 are isomorphic. [2]

3

5 The substitution  $y = u^k$ , where  $k$  is an integer, is to be used to solve the differential equation

$$x \frac{dy}{dx} + 3y = x^2 y^2 \quad (\text{A})$$

by changing it into an equation (B) in the variables  $u$  and  $x$ .

(i) Show that equation (B) may be written in the form

$$\frac{du}{dx} + \frac{3}{kx} u = \frac{1}{k} x u^{k+1}. \quad [4]$$

(ii) Write down the value of  $k$  for which the integrating factor method may be used to solve equation (B). [1]

(iii) Using this value of  $k$ , solve equation (B) and hence find the general solution of equation (A), giving your answer in the form  $y = f(x)$ . [4]

6 (a) The set of polynomials  $\{ax + b\}$ , where  $a, b \in \mathbb{R}$ , is denoted by  $P$ . Assuming that the associativity property holds, prove that  $P$ , under addition, is a group. [4]

(b) The set of polynomials  $\{ax + b\}$ , where  $a, b \in \{0, 1, 2\}$ , is denoted by  $Q$ . It is given that  $Q$ , under addition modulo 3, is a group, denoted by  $(Q, +(\text{mod}3))$ .

(i) State the order of the group. [1]

(ii) Write down the inverse of the element  $2x + 1$ . [1]

(iii)  $q(x) = ax + b$  is any element of  $Q$  other than the identity. Find the order of  $q(x)$  and hence determine whether  $(Q, +(\text{mod}3))$  is a cyclic group. [4]

7 (In this question, the notation  $\Delta ABC$  denotes the area of the triangle  $ABC$ .)

The points  $P, Q$  and  $R$  have position vectors  $p\mathbf{i}, q\mathbf{j}$  and  $r\mathbf{k}$  respectively, relative to the origin  $O$ , where  $p, q$  and  $r$  are positive. The points  $O, P, Q$  and  $R$  are joined to form a tetrahedron.

(i) Draw a sketch of the tetrahedron and write down the values of  $\Delta OPQ, \Delta OQR$  and  $\Delta ORP$ . [3]

(ii) Use the definition of the vector product to show that  $\frac{1}{2} |\overrightarrow{RP} \times \overrightarrow{RQ}| = \Delta PQR$ . [1]

(iii) Show that  $(\Delta OPQ)^2 + (\Delta OQR)^2 + (\Delta ORP)^2 = (\Delta PQR)^2$ . [6]

8 (i) Use de Moivre's theorem to express  $\cos 4\theta$  as a polynomial in  $\cos \theta$ . [4]

(ii) Hence prove that  $\cos 4\theta \cos 2\theta \equiv 16 \cos^6 \theta - 24 \cos^4 \theta + 10 \cos^2 \theta - 1$ . [1]

(iii) Use part (ii) to show that the only roots of the equation  $\cos 4\theta \cos 2\theta = 1$  are  $\theta = n\pi$ , where  $n$  is an integer. [3]

(iv) Show that  $\cos 4\theta \cos 2\theta = -1$  only when  $\cos \theta = 0$ . [3]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.