GCE

Mathematics

Unit **4737**: Decision Mathematics 2

Advanced GCE

Mark Scheme for June 2015

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It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

PREPARATION FOR MARKING ON-SCREEN

1. Make sure that you have accessed and completed the relevant and training packages for on-screen marking: *scoris assessor Online Training* and the *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the Instructions for On-Screen Marking and the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to scoris and mark 5 practice responses ("scripts") and 10 standardisation responses

   YOU MUST MARK 5 PRACTICE AND 10 STANDARDISATION RESPONSES BEFORE YOU CAN BE APPROVED TO MARK LIVE SCRIPTS.

MARKING INSTRUCTIONS

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the scoris 50% and 100% deadlines. If you experience problems, you must contact your Team Leader without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the scoris messaging system, or by email.

5. Work crossed out:
   a. where a candidate crosses out an answer and provides an alternative response, the crossed out response is not marked and gains no marks
   b. if a candidate crosses out an answer to a whole question and makes no second attempt, and if the inclusion of the answer does not cause a rubric infringement, the assessor should attempt to mark the crossed out answer and award marks appropriately.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there then add a tick to confirm that the work has been seen.

There is a NR (No Response) option. Award NR (No Response)
- if there is nothing written at all in the answer space
- OR if there is a comment which does not in anyway relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark) which isn't an attempt at the question

Note: Award 0 marks - for an attempt that earns no credit (including copying out the question)

7. The scoris **comments box** is used by your team leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**
   If you have any questions or comments for your team leader, use the phone, the scoris messaging system, or e-mail.

8. Assistant Examiners will send a brief report on the performance of candidates to your Team Leader (Supervisor) by the end of the marking period. The Assistant Examiner's Report Form (AERF) can be found on the RM Cambridge Assessment Support Portal (and for traditional marking it is in the *Instructions for Examiners*). Your report should contain notes on particular strength displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

9. For answers marked by levels of response:
   a. **To determine the level** – start at the highest level and work down until you reach the level that matches the answer
   b. **To determine the mark within the level**, consider the following:

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Here are the subject specific instructions for this question paper

a   Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an

intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
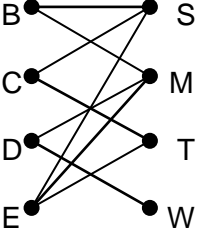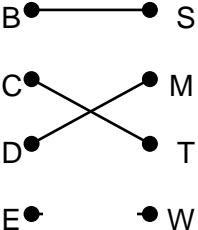
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
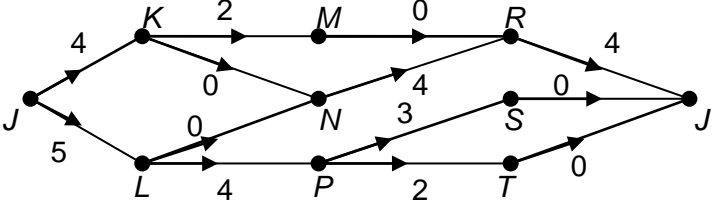
12.   Here is the mark scheme for this question paper.

MARK SCHEME
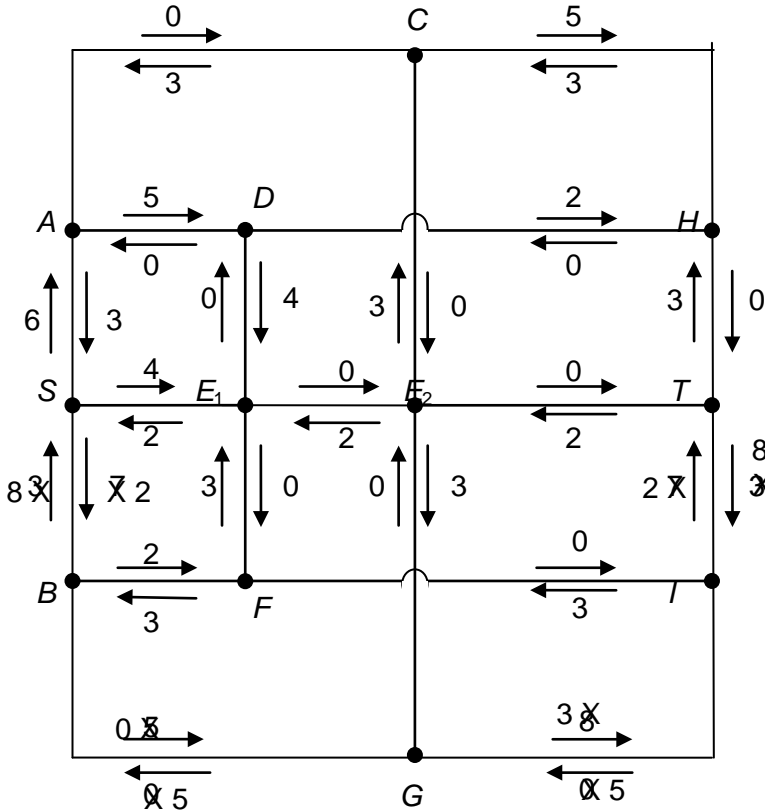
Question			Answer/Indicative content	Mark	Guidance
1	(i)			B1 [1]	Bipartite graph correct Ignore any working shown on diagram, provided bipartite graph can be identified easily
	(ii)			B1 [1]	Incomplete matching correct Ignore any working shown on diagram, provided incomplete matching can be identified easily Do not give credit if only shown superimposed on bipartite graph from part (i)
	(iii)		E = M − D = W Beryl Batty is invited on Sunday Colonel Chapman is invited on Tuesday Dimitri Delacruz is invited in Wednesday Erina El-Sayed is invited on Monday	M1 A1 [2]	This path only, written, condone path in reverse May be implied from correct matching This matching written out (cao) Allow use of S, M, T, W as shorthand for the days
	(iv)		Beryl Batty	B1 [1]	B

2	(i)							

Activity	Duration (hours)	Immediate predecessors
A	4	-
B	5	-
C	4	B
D	2	A
E	4	A, B
F	3	C
G	2	C
H	4	D, E

B1   Precedences correct for *A*, *B*, *C*, *D*
      May leave *A* and *B* blank

B1   Precedences correct for *E*, *F*, *G*, *H*

[2]

(ii)

M1   Forward pass substantially correct ($\leq$ 1 independent error)

M1   Backward pass substantially correct ($\leq$ 1 independent error)

A1   Both passes correct

Critical activities: *B E H*   B1   *B*, *E*, *H* (cao) (written)
Minimum project completion time: 13 hours   B1   13 (units not necessary) (written, not implied from working)

[5]

(iii) | 1 (hour) | B1 | 1 (units not necessary) (cao)

[1]

(iv) (a) | 3 (workers) | B1 | 3 (cao)

[1]

(iv) (b) | Total time = 28 hours = 2x14, so must be at least 14 hours <u>Or</u> *F*, *G* and *H* will clash (no matter how much they are moved about within the windows defined in the diagram) | B1 | Convincingly explaining why 13 hours is not possible with only two people

[1]

(v) | *C* and *F* become critical instead of *E* and *H*
Critical activities are now *B*, *C*, *F* (with delay on *C*) | B1 | Identifying the critical activities as *B*, *C*, *F*
Minimum project completion time becomes 14 hours | B1 | 14 (units not necessary)

[2]

3 (i)

	S	C	R	X
F	30	75	48	100
G	25	82	45	100
H	45	76	53	100
I	40	70	45	100

Reduced by cols (rows by default)

5	5	3	0
0	12	0	0
20	6	8	0
15	0	0	0

Augment by 3

2	2	0	0
0	12	0	3
17	3	5	0
15	0	0	3

Swim = Gary   Cycle = Isobel   Run = Fred
25+70+48 =143 minutes (or 2 hours 23 min)

B1 Add a dummy column of <u>equal</u> (non-negative) values (seen, not implied)

M1 Reducing their matrix by columns (even if not square) (If rows are reduced first, credit this mark when columns are reduced). Condone one numerical error, but not changing to a maximising problem

A1 Reduced cost matrix correct (cao)

B1 Crossing through 0's using min number of lines for their reduced cost matrix (seen, not implied from augmenting)

M1 Substantially correct attempt to augment by minimum uncovered element, from their (<4) lines. Must see at least one cell of each type augmented correctly

A1 Augmentation correct for their reduced cost matrix (must have augmented by the minimum uncovered element, not lots of augmentations by 1)

B1 Correct allocation of people (not times) to stages (cao) May use initial letters, may also say that Helen is the reserve

B1 143 (minutes may be implied) (or 2 hr 23) (cao)

[8]

(ii) 20 minutes

B1 20 (minutes may be implied) (cao)

[1]

(iii)

	S	C	R
F	30	75	48
H	45	76	53
I	40	70	45

eg

0	5	3
15	6	8
10	0	0

0	5	3
9	0	2
10	0	0

Swim = Fred   Cycle = Helen   Run = Isobel

151 minutes

M1 ft 3x3 table (follow through their swimmer removed)

A1 ft Leading to a correct allocation
F removed: s=G   c=H   r=I ⇒ 146
H removed: s=G   c=I   r=F ⇒ 143
I removed: s=G   c=H   r=F ⇒ 149

A1 ft Corresponding time

[3]

4	(i)				

			M1	Structure of graph correct, all vertices labelled appropriately Condone arcs not shown as directed
			A1	Arc weights correct Interpret blanks as 0 Be generous on ambiguous arc weight placings
			B1	Directions on arcs shown (correctly) Condone no direction on arcs with weight 0
				R, S, T joined back to original J $\Rightarrow$ M1, A0, B1 max
			[3]	

(ii)

Using the tables in the question, not following through network errors, column headings need not be as given here

Stage	State	Action	Working	Suboptimal maximum
3	0	0	4	4
	1	0	0	0
	2	0	0	0
2	0	0	0+4 = 4	4
	1	0	4+4 = 8	8
	2	1	3+0 = 3	3
		2	2+0 = 2	
1	0	0	2+4 = 6	
		1	0+8 = 8	8
	1	1	0+8 = 8	8
		2	4+3 = 7	
0	0	0	4+8 = 12	
		1	5+8 = 13	13

(0; 0) − (1; 1) − (2; 1) − (3; 0) − (4; 0)
  J   −   L   −   N   −   R   −   J

13 churches

M1	Stage and state columns correct
A1	Action column correct
B1	Solving a maximum total (longest path) problem (may be implied from working)
B1	Suboptimal values for stage 3 correct, may be seen as working values, allow even if solving min/maximin/minimax
	From here, follow through from previous stage if possible
M1 ft	Working values (totals) correct for stage 2
A1 ft	Suboptimal values correct for stage 2 (ie 4, 8, 3 or ft)
M1 ft	Suboptimal values correct for stage 1 (ie 8, 8 or ft)
A1 ft	Suboptimal value correct for stage 0 (ie 13 or ft)
B1	Route correct, or in reverse, using (stage; state) or letters
B1	13 (do not need to see 'churches') (cao)
[10]	

4   **(ii)**

4	0	0	4	4
		1	0	
		2	0	
3	0	0	4	
		1	8	8
	1	2	3	3
	2	2	2	2
2	0	0	6	6
	1	0	8	8
		1	8	
	2	1	7	7
1	0	0	12	12
	1	0	13	13

Mark	Description
B1	Solving a max problem
	Suboptimal value
B1	stage 4 (ie 4 only)
M1	Working values stage 3
	Suboptimal values
A1	stage 3 (ie 8, 3, 2)
B1	stage 2 (ie 6, 8, 7)
B1	stage 1 (ie 12, 13)
B1, B1	Route and 13
max 8 marks	

This page shows three alternatives that gain partial credit
In each case there will be no follow through (cao)

Check the stage label for the first row of the table and mark accordingly (apart from a spurious stage 4 that does nothing, followed by the table above, which will be marked as above or a spurious stage 0 that does nothing, followed by the table below left, which will be marked as below left)

Any rows missing are penalised by not awarding the appropriate A or B for suboptimal values

1	0	0	4	4
		1	0	5	5
2	0	0	6	6
		1	0	4	
	1	1	5	5
	2	1	9	9
3	0	0	6	
		1	9	9
	1	2	12	12
	2	2	11	11
4	0	0	13	13
		1	12	
		2	11	

Mark	Description
B1	Solving a max problem
	Suboptimal value(s)
B1	stage 1 (ie 4, 5)
B1	stage 2 (ie 6, 5, 9)
B1	stage 3 (ie 9, 12, 11)
B1	stage 4 (ie 13 only)
B1, B1	Route and 13
max 7 marks	

0	0	0	4	
		1	5	5
1	0	0	6	6
		1	4	
	1	1	5	
		2	9	9
2	0	0	6	6
	1	0	9	9
	2	1	12	12
		2	11	
3	0	0	13	13
	1	0	12	12
	2	0	11	11

Mark	Description
B1	Solving a max problem
	Suboptimal values
B1	stages 0 and 1 (ie 5 and 6, 9)
B1	stage 2 (ie 6, 9, 12)
B1	stage 3 (ie 13, 12, 11)
B1, B1	Route and 13
max 6 marks	

5	(i)		3	B1 **[1]**	cao
	(ii)		8	B1 **[1]**	cao
	(iii)		5 (litres per second)	B1	5 (units not necessary)

Augmenting route *SBGIT* by 5 — B1

[2]

		(iv)	Maximum flow is 8+5 = 13 (litres per second)	B1	13 (units not necessary) (cao)
			Cut: $\{S\ A\ B\ C\ D\ E_1\ F\ H\}\ \{E_2\ G\ I\ T\}$ Cut across arcs BG, FI, E_1E_2, CE_2, HT	M1	This cut described in any valid way (do not credit cut being drawn in (iii)) - note: arc E_2G is **not** part of a valid cut
			Value of cut =13 and flow = 13, min cut = max flow	A1	A valid explanation cut =13 and flow = 13 cut = 13 and min cut = max flow cut arcs are saturated so no more can flow
				[3]	
		(v)	A <u>vertex restriction</u> of 2 (litres per second)	B1	<u>At most</u> 2 (litres per second) can flow <u>through</u> E Flow <u>through</u> E $\leq$ 2
				[1]	
		(vi)		B1 B1	Correct network (with E_1 and E_2 shown as a single vertex E) and directed arcs joined to and from E correctly Flow correct, shown as a flow not as labelling procedure For this mark condone directions missing but not arcs missing Need E as a single vertex, not split as E_1 and E_2
				[2]	

6	(i)						M1	Any three entries correct
			1	2	3		A1	This table (cao)
		1	-5	-10	20			May see working for (iii), ignore this here
		2	5	0	-5			
		3	20	10	-25		**[2]**	

	(ii)	-5 when each chooses strategy 1 means that R has five (thousand) fewer soldiers than C	M1	R has 5 fewer than C, C has an excess of 5 (thousand)
		C has 5 (thousand) more than R, -5 + 5 = 0 (and total is 0 for every cell)	A1 **[2]**	C has 5 more than R so total is 0 (no need to say 0 for all cells) or using 15-20 <u>and</u> 20-15 appropriately, or equivalent

	(iii)	Row minima: -10, -5, -25 row maximin = -5	M1	Evidence of using row minima appropriately, may be seen on table from (i), may be implied from -5 or strategy 2 chosen (www)
		Play-safe strategy for R is (strategy) 2	A1	2 (written in the appropriate answer space)
		Col maxima: 20, 10, 20 (or use –negatives of these) col minimax = 10 (or -10)	M1	Evidence of using ($\pm$) col maxima appropriately, may be seen on table from (i), may be implied from 10 or strategy 2 chosen (www)
		Play-safe strategy for C is (strategy) 2	A1	2 (written in the appropriate answer space)
		If R plays safe then C should choose (strategy) 3	B1 ft **[5]**	Follow through their chosen play-safe strategy for R [$R = 1 \Rightarrow C = 2$, $R = 3 \Rightarrow C = 3$]

	(iv)				M1 ft	Reducing to 3 rows and 2 columns (even if wrong column has been removed), or a correct ft from their table in (i)
			2	3	A1	Correct reduced matrix with rows and columns labelled correctly (cao)
		1	-10	20		
		2	0	-5		
		3	10	-25	**[2]**	

	(v)		x, y and z represent the <u>probability</u> of R choosing strategies 1, 2, 3, respectively	B1	'Prob' <u>and</u> 'R' <u>and</u> 'strategies' (or 1, 2 and 3),or equivalent
			Add 25 throughout reduced matrix from (iii), to remove negative entries Then, if C chooses strategy 2 then R can expect $15x+25y+35z$, so the minimum, m must be $\leq$ this	B1 **[2]**	Add 25 <u>and</u> C chooses 2 (first column)
	(vi)		Using random numbers, General Rose should send 1 division (North) (and 3 South) with probability 0.5385 and send 3 divisions (North) (and 1 South) with probability 0.4615	M1 A1	Relating solution to sending 1 and 3 divisions N (and the rest S). Note: need to discuss divisions not just strategies Using probabilities 0.5385 (or 1077/2000) and 0.4615 (or 923/2000) correctly If M0, SC 1 for a correct interpretation (in terms of divisions) of either (non zero) probability
			Optimal $m = 24.23$ (24.2325) so optimal $M = -0.77$ (-0.7675)	B1	-0.77 or anything rounding to -0.77
			R can expect to lose 770 (767, 767.5, 768) soldiers C can expect to win the same number (per game)	B1 **[4]**	Interpretation of -0.77 (for <u>both</u> R and C) Condone R loses 0.77 and C gains 0.77 Need not say 'per game'
	(vii)		If $x = 0$ then $y + 7z = 0$, but y and z are both non-negative (and must total 1)	B1 **[1]**	Showing that this leads to inconsistent values $y = \frac{7}{6}$ but y cannot exceed 1 <u>or</u> $z = -\frac{1}{6}$ but z cannot be < 0
	(viii)		Equations are satisfied, but $M = -\frac{10}{7}$ (-1.43) which is smaller than -0.77	M1 A1 **[2]**	$M = -\frac{10}{7}$ (-1.43) or $m = \frac{165}{7}$ (23.57, or rounded to 23.6) Stating that this is smaller than corresponding (correct) value from (vi), not ft

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