

4726

Mark Scheme

June 2010

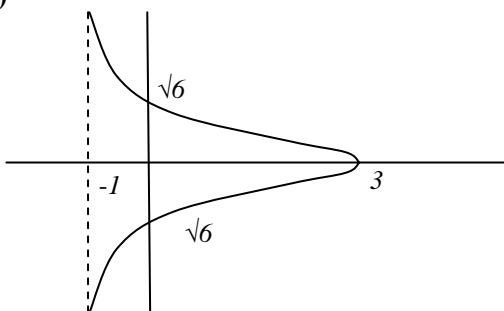
- 1** Derive/quote $g'(x) = p/(1+x^2)$
 Attempt $f'(x)$ as $a/(1+bx^2)$
 Use $x = 1/2$ to set up a solvable equation in p , leading to at least one solution
 Get $p = 5/4$ only
- 2** Reasonable attempt at $e^{2x}(1+2x+2x^2)$
 Multiply out their expressions to get all terms up to x^2
 Get $1+3x+4x^2$
 Use binomial, equate coefficients to get 2 solvable equations in a and n
 Reasonable attempt to eliminate a or n
 Get $n=9, a=1/3$ cwo
- 3** Quote/derive correct $dx=2dt/(1+t^2)$
 Replace all x (not $dx=dt$)
 Get $2/(t-1)^2$ or equivalent
 Reasonable attempt to integrate their expression
 Use correct limits in their correct integral
 Clearly tidy to $\sqrt{3}+1$ from cwo
- 4 (i)** Get $a = -2$
 Get $b = 6$
 Get $c = 1$

- B1
 M1 Allow any $a, b=2$ or 4
 M1
 A1 AEEF
- M1 3 terms of the form $1+2x+ax^2, a \neq 0$
 M1 (3 terms) x (minimum of 2 terms)
 A1 cao
 Reasonable attempt at binomial, each term
 M1 involving a and n ($an=3, a^2n(n-1)/2=4$)
 M1
 A1 cao
 SC Reasonable $f'(x)$ and $f''(x)$ using product rule (2 terms) M1
 Use their expressions to find $f'(0)$ and $f''(0)$ M1
 Get $1+3x+4x^2$ cao A1

- B1
 M1 From their expressions
 A1
 M1
 A1√ Must involve $\sqrt{3}$
 A1 A.G.

- B1 May be quoted
 B1 May be quoted
 B1 May be quoted
- (from correct working)

(ii)



- B1 Correct shape in $-1 < x \leq 3$ only (allow just top or bottom half)
- B1 90° (at $x=3$) (must cross x -axis i.e. symmetry)
- B1 Asymptote at $x = -1$ only (allow -1 seen)
- B1√ Correct crossing points; $\pm\sqrt{(b/c)}$ from their b, c

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- 5 (i)** Reasonable attempt at parts
 Get $e^x(1-2x)^n - \int e^x \cdot n(1-2x)^{n-1} \cdot -2 dx$
 Evidence of limits used in integrated part
 Tidy to A.G.
- M1 Leading to second integral
 A1 Or $(1-2x)^{n+1}/(-2(n+1))e^x - \int (1-2x)^{n+1}/(-2(n+1))e^x dx$
 M1 Should show ± 1
 A1 Allow $I_{n+1} = 2(n+1)I_n - 1$
- (ii)** Show any one of $I_3=6I_2-1$, $I_2=4I_1-1$,
 $I_1=2I_0-1$
 Get $I_0(=e^{1/2}-1)$ or $I_1(=2e^{1/2}-3)$
 Substitute their values back for their I_3
 Get $48e^{1/2} - 79$
- B1 May be implied
 B1
 M1 Not involving n
 A1
- 6 (i)** Reasonable attempt to differentiate
 $\sinh y = x$ to get dy/dx in terms of y
 Replace $\sinh y$ to A.G.
- M1 Allow $\pm \cosh y dy/dx = 1$
 A1 Clearly use $\cosh^2 - \sinh^2 = 1$
 SC Attempt to diff. $y = \ln(x + \sqrt{x^2+1})$
 using chain rule M1
 Clearly tidy to A.G. A1
- (ii)** Reasonable attempt at chain rule
 Get $dy/dx = a \sinh(asinh^{-1}x)/\sqrt{x^2+1}$
 Reasonable attempt at product/quotient
 Get d^2y/dx^2 correctly in some form
 Substitute in and clearly get A.G.
- M1 To give a product
 A1
 M1 Must involve \sinh and \cosh
 A1 $\sqrt{\text{From } dy/dx = k \sinh(asinh^{-1}x)/\sqrt{x^2+1}}$
 A1
 SC Write $\sqrt{x^2+1}dy/dx = k \sinh(asinh^{-1}x)$
 or similar
 Derive the A.G.
- 7 (i)** Get 5.242, 5.239, 5.237
 Get 5.24
- B1 $\sqrt{\text{Any 3(minimum) correct from previous value}}$
 B1 Allow one B1 for 5.24 seen if 2 d.p.used
- (ii)** Show reasonable staircase for any region
 Describe any one of the three cases
 Describe all three cases
- B1 Drawn curve to line
 B1
 B1
- (iii)** Reasonable attempt to use log/expo. rules
 Clearly get A.G.
 Attempt $f'(x)$ and use at least once in
 correct N-R formula
 Get answers that lead to 1.31
- M1 Allow derivation either way
 A1
 M1
 A1 Minimum of 2 answers; allow
 truncation/rounding to at least 3 d.p.
- (iv)** Show $f'(\ln 36) = 0$
 Explain why N-R would not work
- B1
 B1 Tangent parallel to Ox would not meet Ox again
 or divide by 0 gives an error

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- 8 (i)** Use correct definition of $\cosh x$ B1
 Attempt to cube their definition M1
 involving e^x and e^{-x} (or e^{2x} and e^x) M1 Must be 4 terms
 Put their 4 terms into LHS and attempt M1
 to simplify A1
 Clearly get A.G. SC Allow one B1 for correct derivation from
cosh3x = cosh(2x+x)
- (ii)** Rewrite as $k\cosh 3x = 13$ M1
 Use \ln equivalent on $13/k$ M1 Allow $\pm \ln$ or $\ln(13/k \pm \sqrt{(13/k)^2 - 1})$ for their k
or attempt to set up and solve quadratic via
exponentials
- Get $x = (\pm) \frac{1}{3} \ln 5$ A1
 Replace in $\cosh x$ for u M1
 Use $e^{aln b} = b^a$ at least once M1
 Get $\frac{1}{2}(5^{1/3} + 5^{-1/3})$ A1
- 9 (i)** Attempt integral as $k(2x+1)^{1.5}$ M1
 Get 9 A1 cao
 Attempt subtraction of areas M1 Their answer – triangle
 Get 3 A1√ Their answer – 6 (>0)
- (ii)** Use $r^2 = x^2 + y^2$ and $x = r\cos\theta, y = r\sin\theta$ B1
 Eliminate x and y to produce quadratic M1
 equation ($=0$) in r (or $\cos\theta$)
 Solve their quadratic to get r in terms of θ A1√
 (or vice versa)
 Clearly get A.G. A1 $r > 0$ may be assumed
- Clearly show θ_1 (at B) = $\tan^{-1} \frac{3}{4}$ and B1
 θ_2 (at A) = π SC Eliminate y to get r in terms of x only M1
Get $r = x + 1$ A1
SC Start with $r = 1/(1 - \cos\theta)$ and derive cartesian
- (iii)** Use area = $\frac{1}{2} \int r^2 d\theta$ with correct r B1 cwo; ignore limits
 Rewrite as $k\operatorname{cosec}^4(\frac{1}{2}\theta)$ M1 Not just quoted
 Equate to their part (i) and tidy M1 To get $\int =$ some constant
 Get 24 A1 A.G.