Mark Scheme 4726
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1		Correct formula with correct r	M1	Allow $r^2 = 2\sin^2 3\theta$
		Rewrite as $a + b\cos 6\theta$	M1	$a, b \neq 0$
		Integrate their expression correctly	A1√	From $a + b\cos 6\theta$
		Get $\frac{1}{3}\pi$	A1	cao
2	(i)	Expand to $\sin 2x \cos\frac{1}{4}\pi + \cos 2x \sin\frac{1}{4}\pi$	B1	
		Clearly replace $\cos\frac{1}{4}\pi$, $\sin\frac{1}{4}\pi$ to A.G.	B1	
	(ii)	Attempt to expand $\cos 2x$	M1	Allow $1 - 2x^2/2$
		Attempt to expand $\sin 2x$	M1	Allow $2x - 2x^3/3$
		Get $\frac{1}{2}\sqrt{2}\,(1 + 2x - 2x^2 - 4x^3/3)$	A1	Four correct unsimplified terms in any order; allow bracket; AEEF SR Reasonable attempt at $f^{n}(0)$ for $n = 0$ to 3 M1 Attempt to replace their values in Maclaurin M1 Get correct answer only A1
3	(i)	Express as $A/(x-1) + (Bx+C)/(x^2+9)$	M1	Allow $C=0$ here
		Equate (x^2+9x) to $A(x^2+9) + (Bx+C)(x-1)$	M1√	May imply above line; on their P.F.
		Sub. for x or equate coeff.	M1	Must lead to at least 3 coeff.; allow cover-up method for A
		Get A=1, B=0, C=9	A1	cao from correct method
	(ii)	Get $A\ln(x-1)$	B1√	On their A
		Get $C/3\tan^{-1}(x/3)$	B1√	On their C; condone no constant; ignore any $B \neq 0$
4	(i)	Reasonable attempt at product rule	M1	Two terms seen
		Derive or quote diff. of $\cos^{-1}x$	M1	Allow +
		Get $-x^2(1-x^2)^{-\frac{1}{2}} + (1-x^2)^{\frac{1}{2}} + (1-x^2)^{-\frac{1}{2}}$	A1	
		Tidy to $2(1-x^2)^{\frac{1}{2}}$	A1	cao
	(ii)	Write down integral from (i)	B1	On any $k\sqrt{(1-x^2)}$
		Use limits correctly	M1	In any reasonable integral
		Tidy to $\frac{1}{2}\pi$	A1	
			SR	Reasonable sub. B1 Replace for new variable and attempt to integrate (ignore limits) M1 Clearly get $\frac{1}{2}\pi$ A1

5	(i)	Attempt at parts on $\int 1\,(\ln x)^n\,dx$	M1	Two terms seen
		Get $x(\ln x)^n - \int n\,(\ln x)^{n-1}\,dx$	A1	
		Put in limits correctly in line above	M1	
		Clearly get A.G.	A1	ln e =1, ln1 =0 seen or implied
	(ii)	Attempt I_3 to I_2 as $I_3 = e - 3I_2$	M1	
		Continue sequence in terms of In	A1	$I_2 = e - 2I_1$ and/or $I_1 = e - I_0$
		Attempt I_0 or I_1	M1	($I_0 = e-1$, $I_1 = 1$)
		Get 6 – 2e	A1	cao
6	(i)	Area under graph (= $\int 1/x^2\,dx$, 1 to n+1) < Sum of rectangles (from 1 to n)	B1	Sum (total) seen or implied eg diagram; accept areas (of rectangles)
		Area of each rectangle = Width x Height = 1 x $1/x^2$	B1	Some evidence of area worked out – seen or implied
	(ii)	Indication of new set of rectangles	B1	
		Similarly, area under graph from 1 to n > sum of areas of rectangles from 2 to n	B1	Sum (total) seen or implied
		Clear explanation of A.G.	B1	Diagram; use of left-shift of previous areas
	(iii)	Show complete integrations of RHS, using correct, different limits	M1	Reasonable attempt at $\int x^{-2}\,dx$
		Correct answer, using limits, to one integral	A1	
		Add 1 to their second integral to get complete series	M1	
		Clearly arrive at A.G.	A1	
	(iv)	Get one limit	B1	Quotable
		Get both 1 and 2	B1	Quotable; limits only required

7	(i)	Use correct definition of cosh or sinh x	B1	Seen anywhere in (i)
		Attempt to mult. their cosh/sinh	M1	
		Correctly mult. out and tidy	A1√	
		Clearly arrive at A.G.	A1	Accept e^{x-y} and e^{y-x}
	(ii)	Get $\cosh(x-y) = 1$	M1	
		Get or imply $(x-y) = 0$ to A.G.	A1	
	(iii)	Use $\cosh^2 x = 9$ or $\sinh^2 x = 8$	B1	
		Attempt to solve cosh x = 3 (not –3) or $\sinh x = \pm\sqrt{8}$ (allow $+\sqrt{8}$ or $-\sqrt{8}$ only)	M1	$x = \ln(3 + \sqrt{8})$ from formulae book or from basic cosh definition
		Get at least one x solution correct	A1	
		Get both solutions correct, x and y	A1	$x, y = \ln(3 \pm 2\sqrt{2})$; AEEF
				SR Attempt tanh = sinh/cosh B1
				Get $\tanh x = \pm\sqrt{8}/3$ (+ or -) M1
				Get at least one sol. correct A1
				Get both solutions correct A1
				SR Use exponential definition B1
				Get quadratic in e^x or e^{2x} M1
				Solve for one correct x A1
				Get both solutions, x and y A1
8	(i)	$x_2 = 0.1890$	B1	
		$x_3 = 0.2087$	B1√	From their x_1 (or any other correct)
		$x_4 = 0.2050$	B1√	Get at least two others correct, all to a minimum of 4 d.p.
		$x_5 = 0.2057$		
		$x_6 = 0.2055$		
		$x_7\ (= x_8) = 0.2056$ (to x_7 minimum)		
		$\alpha = 0.2056$	B1	cao; answer may be retrieved despite some errors
	(ii)	Attempt to diff. $f(x)$	M1	$k/(2+x)^3$
		Use α to show $f'(\alpha) \neq 0$	A1√	Clearly seen, or explain $k/(2+x)^3 \neq 0$ as $k \neq 0$; allow $\pm$ 0.1864
			SR	Translate $y=1/x^2$ M1
				State/show $y=1/x^2$ has no TP A1
	(iii)	$\delta_3 = -0.0037$ (allow –0.004)	B1√	Allow $\pm$, from their x_4 and x_3
	(iv)	Develop from $\delta_{10} = f'(\alpha)\,\delta_9$ etc. to get δ_i or quote $\delta_{10} = \delta_3\, f'(\alpha)^7$	M1	Or any δ_i eg use $\delta_9 = x_{10} - x_9$
		Use their δ_i and $f'(\alpha)$	M1	
		Get 0.000000028	A1	Or answer that rounds to $\pm$ 0.00000003

9	(i)	Quote $x = a$	B1	
		Attempt to divide out	M1	Allow M1 for y=x here; allow
			A1	$(x\text{-}a) + k/(x\text{-}a)$ seen or implied
		Get $y = x - a$	A1	Must be equations
	(ii)	Attempt at quad. in x (=0)	M1	
		Use $b^2 - 4ac \geq 0$ for real x	M1	Allow >
		Get $y^2 + 4a^2 \geq 0$	A1	
		State/show their quad. is always >0	B1	Allow ≥
	(iii)		B1√	Two asymptotes from (i) (need not be labelled)
			B1	Both crossing points
			B1√	Approaches – correct shape
			SR	Attempt diff. by quotient/product rule M1
				Get quadratic in x for $dy/dx = 0$ and note $b^2 - 4ac < 0$ A1
				Consider horizontal asymptotes B1
				Fully justify answer B1