Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MPC2

Unit Pure Core 2

Monday 24 May 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

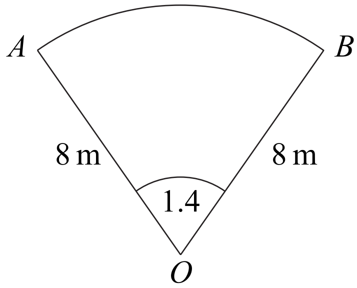
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 8 m and the angle AOB is 1.4 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the perimeter of the sector OAB. *(3 marks)*

(ii) The perimeter of the sector OAB is equal to the circumference of a circle of radius x m. Calculate the value of x to three significant figures. *(2 marks)*

QUESTION PART REFERENCE

2 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = 6 + \frac{2}{5}u_n$$

The first term of the sequence is given by $u_1 = 2$.

(a) Find the value of u_2 and the value of u_3. *(2 marks)*

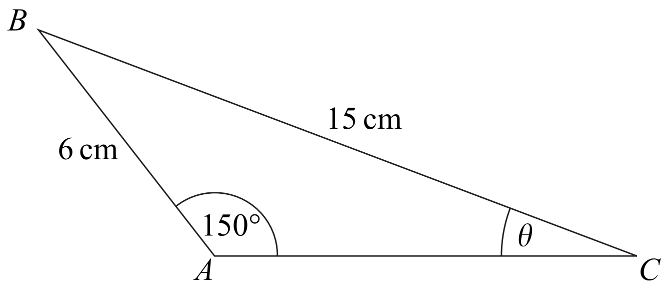
(b) The limit of u_n as n tends to infinity is L.

Write down an equation for L and hence find the value of L. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

3 The triangle ABC, shown in the diagram, is such that $AB = 6\text{ cm}$, $BC = 15\text{ cm}$, angle $BAC = 150°$ and angle $ACB = \theta$.



(a) Show that $\theta = 11.5°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

QUESTION PART REFERENCE

4 (a) The expression $\left(1-\frac{1}{x^2}\right)^3$ can be written in the form

$$1+\frac{p}{x^2}+\frac{q}{x^4}-\frac{1}{x^6}$$

Find the values of the integers p and q. *(2 marks)*

(b) (i) Hence find $\int\left(1-\frac{1}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\int_{\frac{1}{2}}^{1}\left(1-\frac{1}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

QUESTION PART REFERENCE

5 (a) An infinite geometric series has common ratio r.

The first term of the series is 10 and its sum to infinity is 50.

(i) Show that $r = \frac{4}{5}$. *(2 marks)*

(ii) Find the second term of the series. *(2 marks)*

(b) The first and second terms of the geometric series in part **(a)** have the same values as the 4th and 8th terms respectively of an arithmetic series.

(i) Find the common difference of the arithmetic series. *(3 marks)*

(ii) The nth term of the arithmetic series is u_n. Find the value of $\sum_{n=1}^{40} u_n$. *(4 marks)*

QUESTION PART REFERENCE

6 A curve C has the equation

$$y = \frac{x^3 + \sqrt{x}}{x}, \quad x > 0$$

(a) Express $\dfrac{x^3 + \sqrt{x}}{x}$ in the form $x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the normal to the curve C at the point on the curve where $x = 1$. *(4 marks)*

(c) (i) Find $\dfrac{d^2y}{dx^2}$. *(2 marks)*

(ii) Hence deduce that the curve C has no maximum points. *(2 marks)*

QUESTION PART REFERENCE

7 (a) Sketch the graph of $y = \cos x$ in the interval $0 \leqslant x \leqslant 2\pi$. State the values of the intercepts with the coordinate axes. *(2 marks)*

(b) (i) Given that

$$\sin^2\theta = \cos\theta(2 - \cos\theta)$$

prove that $\cos\theta = \frac{1}{2}$. *(2 marks)*

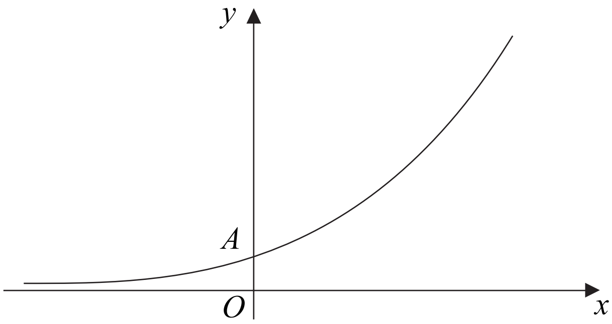
(ii) Hence solve the equation

$$\sin^2 2x = \cos 2x(2 - \cos 2x)$$

in the interval $0 \leqslant x \leqslant \pi$, giving your answers in radians to three significant figures. *(4 marks)*

QUESTION PART REFERENCE

8 The diagram shows a sketch of the curve $y = 2^{4x}$.



The curve intersects the y-axis at the point A.

(a) Find the value of the y-coordinate of A. *(1 mark)*

(b) Use the trapezium rule with six ordinates (five strips) to find an approximate value for $\int_0^1 2^{4x}\,dx$, giving your answer to two decimal places. *(4 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = 2^{4x}$ onto the graph of $y = 2^{4x-3}$. *(2 marks)*

(d) The curve $y = 2^{4x}$ is translated by the vector $\begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$ to give the curve $y = \mathrm{g}(x)$.

The curve $y = \mathrm{g}(x)$ crosses the x-axis at the point Q. Find the x-coordinate of Q. *(4 marks)*

(e) (i) Given that

$$\log_a k = 3\log_a 2 + \log_a 5 - \log_a 4$$

show that $k = 10$. *(3 marks)*

(ii) The line $y = \frac{5}{4}$ crosses the curve $y = 2^{4x-3}$ at the point P. Show that the x-coordinate of P is $\dfrac{1}{4\log_{10} 2}$. *(3 marks)*

QUESTION PART REFERENCE