Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
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7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MFP1

Unit Further Pure 1

Friday 17 May 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The equation

$$x^3 - x^2 + 4x - 900 = 0$$

has exactly one real root, α.

Taking $x_1 = 10$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to four significant figures.

(3 marks)

QUESTION PART REFERENCE

Answer space for question 1

2 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} p & 2 \\ 4 & p \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$$

(a) Find, in terms of p, the matrices:

(i) $\mathbf{A} - \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{AB}$. *(2 marks)*

(b) Show that there is a value of p for which $\mathbf{A} - \mathbf{B} + \mathbf{AB} = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix, and state the corresponding value of k. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

3 (a) Find the general solution, in degrees, of the equation

$$\cos(5x + 40°) = \cos 65°$$

(5 marks)

(b) Given that

$$\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

express $\sin\frac{\pi}{12}$ in the form $\left(\cos\frac{\pi}{4}\right)\left(\cos(a\pi) + \cos(b\pi)\right)$, where a and b are rational.

(3 marks)

QUESTION PART REFERENCE

Answer space for question 3

4 (a) It is given that $z = x + y\mathrm{i}$, where x and y are real numbers.

(i) Write down, in terms of x and y, an expression for $(z - 2\mathrm{i})^*$. *(1 mark)*

(ii) Solve the equation

$$(z - 2\mathrm{i})^* = 4\mathrm{i}z + 3$$

giving your answer in the form $a + b\mathrm{i}$. *(5 marks)*

(b) It is given that $p + q\mathrm{i}$, where p and q are real numbers, is a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$.

Without finding the values of p and q, **state** why $p - q\mathrm{i}$ is **not** a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

5 (a) A curve has equation $y = 2x^2 - 5x$.

The point P on the curve has coordinates $(1, -3)$.

The point Q on the curve has x-coordinate $1 + h$.

(i) Show that the gradient of the line PQ is $2h - 1$. *(3 marks)*

(ii) Explain how the result of part **(a)(i)** can be used to show that the tangent to the curve at the point P is parallel to the line $x + y = 0$. *(2 marks)*

(b) For the improper integral $\int_1^{\infty} x^{-4}(2x^2 - 5x)\,dx$, either show that the integral has a finite value and state its value, or explain why the integral does not have a finite value. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

6 The equation

$$2x^2 + 3x - 6 = 0$$

has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(b) Hence show that $\alpha^3 + \beta^3 = -\frac{135}{8}$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, whose roots are $\alpha + \frac{\alpha}{\beta^2}$ and $\beta + \frac{\beta}{\alpha^2}$. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 6

7 (a) Show that the equation $4x^3 - x - 540\,000 = 0$ has a root, α, in the interval $51 < \alpha < 52$. *(2 marks)*

(b) It is given that $S_n = \sum_{r=1}^{n} (2r-1)^2$.

(i) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = \frac{n}{3}(kn^2 - 1)$, where k is an integer to be found. *(5 marks)*

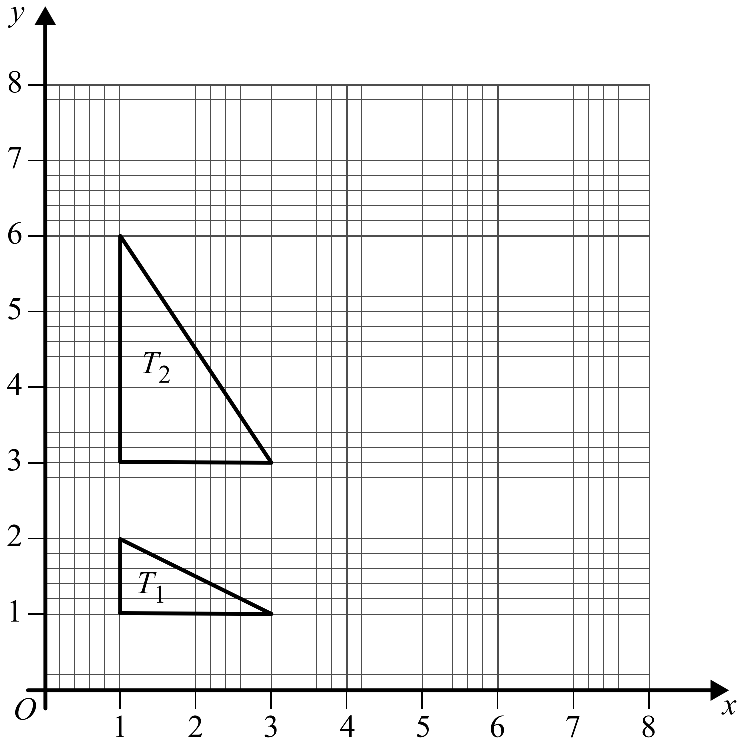
(ii) Hence show that $6S_n$ can be written as the product of three consecutive integers. *(2 marks)*

(c) Find the smallest value of N for which the sum of the squares of the first N odd numbers is greater than $180\,000$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 7

8 The diagram shows two triangles, T_1 and T_2.



(a) Find the matrix which represents the stretch that maps triangle T_1 onto triangle T_2.
(2 marks)

(b) The triangle T_2 is reflected in the line $y = \sqrt{3}x$ to give a third triangle, T_3. Find, using surd forms where appropriate:

(i) the matrix which represents the reflection that maps triangle T_2 onto triangle T_3;
(2 marks)

(ii) the matrix which represents the combined transformation that maps triangle T_1 onto triangle T_3.
(2 marks)

QUESTION PART REFERENCE	**Answer space for question 8**

9 A curve has equation

$$y = \frac{x^2 - 2x + 1}{x^2 - 2x - 3}$$

(a) Find the equations of the three asymptotes of the curve. *(3 marks)*

(b) (i) Show that if the line $y = k$ intersects the curve then

$$(k-1)x^2 - 2(k-1)x - (3k+1) = 0$$ *(1 mark)*

(ii) Given that the equation $(k-1)x^2 - 2(k-1)x - (3k+1) = 0$ has real roots, show that

$$k^2 - k \geqslant 0$$ *(3 marks)*

(iii) Hence show that the curve has only one stationary point and find its coordinates.

(No credit will be given for solutions based on differentiation.) *(4 marks)*

(c) Sketch the curve and its asymptotes. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 9