

# Mark Scheme 4725 January 2006

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|    |   |  |                                       |   |
|----|---|--|---------------------------------------|---|
| 1. | (i) $\frac{2 + 16i - i - 8i^2}{10 + 15i}$<br>(ii)<br>$\frac{1}{5}(10 + 15i)$ or $2 + 3i$  | M1<br>A1<br>M1<br>A1<br><br>A1ft               | 2<br><br>3<br><br><b>5</b>            | Attempt to multiply correctly<br>Obtain correct answer<br>Multiply numerator & denominator by conjugate<br>Obtain denominator 5<br><br>Their part (i) or $10 + 15i$ derived again / 5   |
| 2. | $1^2 = \frac{1}{6} \times 1 \times 2 \times 3$<br>$\frac{1}{6}n(n+1)(2n+1) + (n+1)^2$<br>$\frac{1}{6}(n+1)(n+2)\{2(n+1)+1\}$  | B1<br><br>M1<br>DM1<br><br>A1<br><br>A1        | <br><br><br><br>5<br><br><b>5</b>     | Show result true for $n = 1$ or $2$<br><br>Add next term to given sum formula, any letter OK<br>Attempt to factorise or expand and simplify<br><br>Correct expression obtained<br><br>Specific statement of induction conclusion, with no errors seen |
| 3. | (i)<br>$2 \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1 \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$<br>$2 \times 5 - 1 \times 2 + 3 \times -1$<br>5<br>(ii) | M1<br><br>A1<br>A1<br>B1ft                     | <br><br>3<br><br>1<br><br><b>4</b>    | Show correct expansion process, allow sign slips<br><br>Obtain correct (unsimplified) expression<br>Obtain correct answer<br>State that <b>M</b> is non-singular as $\det \mathbf{M}$ non-zero, ft their determinant                                  |
| 4. | $u^2 + 4u + 4$<br>$u^3 + 6u^2 + 12u + 8$<br><br>$u = \sqrt[3]{5}$<br>$x = 2 + \sqrt[3]{5}$  | B1<br><br><br>M1<br>A1<br><br>A1ft<br><br>A1ft | <br><br><br><br><br>5<br><br><b>5</b> | $u + 2$ squared and cubed correctly<br><br>Substitute these and attempt to simplify<br>Obtain $u^3 - 5 = 0$ or equivalent<br><br>Correct solution to their equation<br><br>Obtain 2 + their answer<br>[Decimals score 0/2 of final A marks]           |

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| 5. | $8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$ | M1 | 6 | Consider the sum of three separate terms    |
|    | $8\Sigma r^3 = 2n^2(n+1)^2$             | A1 |   | Correct formula stated or used a.e.f.       |
|    | $6\Sigma r^2 = n(n+1)(2n+1)$            | A1 |   | Correct formula stated or used a.e.f.       |
|    | $2\Sigma r = n(n+1)$                    | A1 |   | Correct term seen                           |
|    | $2n^3(n+1)$                             | M1 |   | Attempt to factorise or expand and simplify |
|    | <b>AG</b>                               | A1 |   | Obtain given answer correctly               |

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| 6.   | (i) $\frac{1}{2} \begin{pmatrix} 8 & -2 \\ -3 & 1 \end{pmatrix}$                 | B1                                       | 2        | Transpose leading diagonal and negate other diagonal  |
|  | (ii) Either  | B1                                       |          | Divide by determinant   |
|  | $\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$                     | M1A1                                     | 5        | State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$                                    |
|  | Or   | M1                                       |          | Use this result and obtain $\mathbf{B}^{-1} = \mathbf{C}^{-1}\mathbf{A}$ , or equivalent matrix algebra |
|  | $\frac{1}{5} \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$                     | A1ft                                     |          | Matrix multn., two elements correct, for any pair   |
|  | $\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$   | B1                                       |          | All elements correct ft their (i)   |
|  | $\mathbf{B} = \frac{1}{5} \begin{pmatrix} 0 & -2 \\ 5 & 14 \end{pmatrix}$        | M1                                       |          | Find $\mathbf{A}^{-1}$  |
|  | $\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$                     | M1                                       |          | Premultiply by $\mathbf{A}^{-1}$ stated or implied  |
|  | Or   | A1ft                                     |          | Matrix multn. Two elements correct  |
|  | $\mathbf{AB} = \begin{pmatrix} 2a + c & 2b + d \\ a + 3c & b + 3d \end{pmatrix}$ | A1                                       |          | All elements correct  |
| $a = 0, c = 1, b = -0.4, d = 2.8$                            | B1   | Correct $\mathbf{B}^{-1}$                |          |   |
| $\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$ | M1   | Find $\mathbf{AB}$                       |          |   |
|  | A1A1   | Solve one pair of simultaneous equations |          |   |
|  | A1   | Each pair of answers                     |          |   |
|  |  | Correct $\mathbf{B}^{-1}$                | <b>7</b> |   |

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| 7. | <p>(a) (i) <math>\sqrt{13}</math></p> <p>(ii)</p> <p>- 0.59</p> <p>(b)</p> <p><math>1 - 2i</math></p> <p>(c)</p>  | <p>B1</p> <p>M1<br/>A1<br/>A1</p> <p>M1</p> <p>A1A1<br/>A1</p> <p>B1<br/>B1</p>  | <p>1</p> <p>3</p> <p>4</p> <p>2</p> <p><b>10</b></p> | <p>Obtain correct answer, decimals OK</p> <p>Using <math>\tan^{-1}b/a</math>, or equivalent trig allow + or -<br/>Obtain 0.59</p> <p>Obtain correct answer</p> <p>Express LHS in Cartesian form &amp; equate real and imaginary parts<br/>Obtain <math>x = 1</math> and <math>y = -2</math></p> <p>Correct answer written as a complex number</p> <p>Sketch of vertical straight line<br/>Through <math>(-0.5, 0)</math></p>                 |
| 8. | <p>(i)</p> <p><math>\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix}</math></p> <p>(ii) Either <math>\begin{pmatrix} 1 &amp; 0 \\ 0 &amp; -1 \end{pmatrix}</math></p> <p><math>\begin{pmatrix} 2 &amp; 0 \\ 0 &amp; 2 \end{pmatrix}</math></p> <p>Or <math>\begin{pmatrix} -1 &amp; 0 \\ 0 &amp; 1 \end{pmatrix}</math></p> <p><math>\begin{pmatrix} -2 &amp; 0 \\ 0 &amp; -2 \end{pmatrix}</math></p> <p>Or <math>\begin{pmatrix} 2 &amp; 0 \\ 0 &amp; 1 \end{pmatrix}</math></p> <p><math>\begin{pmatrix} 1 &amp; 0 \\ 0 &amp; -2 \end{pmatrix}</math></p> | <p>B1</p> <p>B1<br/>B1</p> <p>B1,B1<br/>B1</p> <p>B1,B1<br/>B1</p> <p>B1,B1<br/>B1</p> <p>B1,B1<br/>B1</p> <p>B1,B1<br/>B1</p> <p>B1,B1<br/>B1</p> | <p>3</p> <p>6</p> <p>6</p> <p>9</p>                  | <p>For correct vertex <math>(2, -2)</math></p> <p>For all vertices correct<br/>For correct diagram</p> <p>Reflection, in x-axis<br/>Correct matrix</p> <p>Enlargement, centre O s.f. 2<br/>Correct matrix</p> <p>Reflection, in the y-axis<br/>Correct matrix</p> <p>Enlargement, centre O s.f. -2<br/>Correct matrix</p> <p>Stretch, in x-direction s.f. 2<br/>Correct matrix</p> <p>Stretch, in y-direction s.f. -2<br/>Correct matrix</p> |

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| 9.  | <p>(i) <math>\frac{r+2-r}{r(r+2)}</math><br/><math>\frac{2}{r(r+2)}</math></p> <p style="text-align: center;"><b>AG</b></p> <p>(ii)</p> $\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}$ <p>(iii) (a)</p> $\frac{3}{2}$ <p>(b)</p> $\frac{1}{n+1} + \frac{1}{n+2}$   | <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1ft</p> <p>M1</p> <p>A1 ft</p>  | <p>2</p> <p>5</p> <p>1</p> <p>2</p> <p><b>10</b></p> | <p>Show correct process for subtracting fractions</p> <p>Obtain given answer correctly</p> <p>Express terms as differences using (i)</p> <p>Express 1<sup>st</sup> 3 (or last 3) terms so that cancelling occurs</p> <p>Obtain <math>1 + \frac{1}{2}</math></p> <p>Obtain <math>-\frac{1}{n+2}, -\frac{1}{n+1}</math></p> <p>Obtain correct answer in any form</p> <p>Obtain value from their sum to <math>n</math> terms</p> <p>Using (iii) (a) – (ii) or method of differences again<br/>[ <math>n \rightarrow \infty</math> is a method error ]</p> <p>Obtain answer in any form</p>  |
| 10. | <p>(i) <math>\alpha + \beta + \gamma = 9</math></p> <p>(ii)</p> $p = \frac{9 - \alpha}{2}$ <p>(iii) <math>\alpha\beta\gamma = 29</math></p> <p>(iv)</p> $\alpha(p^2 + q^2) = 29$ $q = \sqrt{\frac{29}{\alpha} - \frac{(9 - \alpha)^2}{4}}$ <p>(iv) Alternative method</p> $2p\alpha + p^2 + q^2 = 27$ $q = \sqrt{27 - \frac{(9 - \alpha)^2}{4} - \alpha(9 - \alpha)}$ | <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> | <p>1</p> <p>4</p> <p>1</p> <p>5</p> <p><b>11</b></p> | <p>State or use other root is <math>p - iq</math></p> <p>Substitute into (i)</p> <p>Obtain <math>2p + \alpha = 9</math></p> <p>Obtain correct answer a.e.f.</p> <p>Substitute into (iii)</p> <p>Obtain unsimplified expression with no <math>i</math>'s</p> <p>Rearrange to obtain <math>q</math> or <math>q^2</math></p> <p>Substitute their expression for <math>p</math> a.e.f.</p> <p>Obtain correct answer a.e.f.</p> <p>Substitute into <math>\alpha\beta + \beta\gamma + \gamma\alpha = 27</math></p> <p>Obtain unsimplified expression with no <math>i</math>'s</p> <p>Rearrange to obtain <math>q</math> or <math>q^2</math></p> <p>Substitute their expression for <math>p</math> a.e.f.</p> <p>Obtain correct answer a.e.f.</p> |