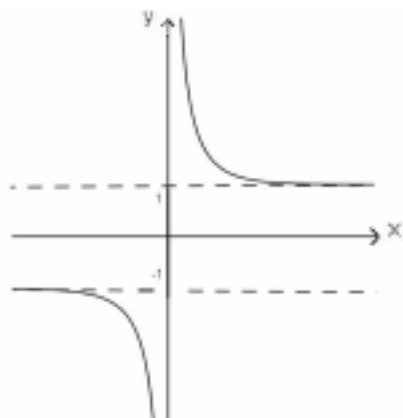


4726 Further Pure Mathematics 2

1(i)	Attempt area = $\pm \Sigma(0.3y)$ for at least three y values Get 1.313(1..) or 1.314	M1 A1	May be implied Or greater accuracy
(ii)	Attempt \pm sum of areas (4 or 5 values) Get 0.518(4..)	M1 A1	May be implied Or greater accuracy SC If answers only seen, 1.313(1..) or 1.314 B2 0.518(4..) B2 -1.313(1..) or -1.314 B1 -0.518(4..) B1
	Or Attempt answer to part (i)–final rectangle Get 0.518(4..)	M1 A1	
(iii)	Decrease width of strips	B1	Use more strips or equivalent
2	Attempt to set up quadratic in x Get $x^2(y-1) - x(2y+1) + (y-1)=0$ Use $b^2 \geq 4ac$ for real x on their quadratic Clearly solve to AG	M1 A1 M1 A1	Must be quadratic; = 0 may be implied Allow =, >, <, \leq here; may be implied If other (in)equalities used, the step to AG must be clear SC Reasonable attempt to diff. using prod/quot rule M1 Solve correct $dy/dx=0$ to get $x=-1, y=1/4$ A1 Attempt to justify inequality e.g. graph or to show $d^2y/dx^2 > 0$ M1 Clearly solve to AG A1
3(i)	Reasonable attempt at chain rule Reasonable attempt at product/quotient rule Correctly get $f'(0) = 1$ Correctly get $f''(0) = 1$	M1 M1 A1 A1	Product in answer Sum of two parts SC Use of $\ln y = \sin x$ follows same scheme
(ii)	Reasonable attempt at Maclaurin with their values Get $1 + x + \frac{1}{2}x^2$	M1 A1 \checkmark	In $af(0) + bf'(0)x + cf''(0)x^2$ From their $f(0), f'(0), f''(0)$ in a correct Maclaurin; all non-zero terms
4	Attempt to divide out. Get $x^3 = A(x-2)(x^2+4) + B(x^2+4) + (Cx+D)(x-2)$ State/derive/quote $A=1$ Use x values and/or equate coeff	M1 M1 A1 M1	Or $A+B/(x-2) + (Cx+D)/(x^2+4)$; allow $A=1$ and/or $B=1$ quoted Allow \checkmark mark from their Part Fract; allow $D=0$ but not $C=0$ To potentially get all their constants

	Get $B=1, C=1, D=-2$	A1	For one other correct from cwo
		A1	For all correct from cwo
5(i)	Derive/quote $d\theta=2dt/(1+t^2)$	B1	May be implied
	Replace their $\cos \theta$ and their $d\theta$, both in terms of t	M1	Not $d\theta = dt$
	Clearly get $\int(1-t^2)/(1+t^2) dt$ or equiv	A1	Accept limits of t quoted here
	Attempt to divide out	M1	Or use AG to get answer above
	Clearly get/derive AG	A1	
		SC	
		Derive $d\theta = 2\cos^2\frac{1}{2}\theta dt$	B1
		Replace $\cos\theta$ in terms of half-angles and their $d\theta$ ($\neq dt$)	M1
		Get $\int 2\cos^2\frac{1}{2}\theta - 1 dt$ or	
		$\int 1 - 1/2\cos^2\frac{1}{2}\theta .2/(1+t^2) dt$	A1
		Use $\sec^2\frac{1}{2}\theta = 1+t^2$	M1
		Clearly get/derive AG	A1
(ii)	Integrate to $a\tan^{-1}bt - t$	M1	
	Get $1/2\pi - 1$	A1	
6	Get $k \sinh^{-1}k_1x$	M1	For either integral; allow attempt at ln version here
	Get $1/3 \sinh^{-1}3/4x$	A1	Or ln version
	Get $1/2 \sinh^{-1}2/3x$	A1	Or ln version
	Use limits in their answers	M1	
	Attempt to use correct ln laws to set up a solvable equation in a	M1	
	Get $a = 2^{1/3} \cdot 3^{1/2}$	A1	Or equivalent

7(i)



- B1 y-axis asymptote; equation may be implied if clear
- B1 Shape
- B1 $y = \pm 1$ asymptotes; may be implied if seen as on graph

(ii) Reasonable attempt at product rule, giving two terms

M1

Use correct Newton-Raphson at least once with their $f'(x)$ to produce an x_2

M1

May be implied

Get $x_2 = 2.0651$

A1√

One correct at any stage if reasonable

Get $x_3 = 2.0653, x_4 = 2.0653$

A1

cao; or greater accuracy which rounds

(iii) Clearly derive $\coth x = 1/2x$

B1

AG; allow derivation from AG
Two roots only

Attempt to find second root e.g. symmetry

M1

Get ± 2.0653

\pm their iteration in part (ii)

A1√

8(i) (a) Get $1/2(e^{\ln a} + e^{-\ln a})$
Use $e^{\ln a} = a$ and $e^{-\ln a} = 1/a$
Clearly derive AG

M1

M1

A1

(b) Reasonable attempt to multiply out their attempts at exponential definitions of cosh and sinh

M1

4 terms in each

Correct expansion seen as $e^{(x+y)}$ etc.

A1

Clearly tidy to AG

A1

With $e^{-(x-y)}$ seen or implied

(ii) Use $x = y$ and $\cosh 0 = 1$ to get AG

B1

(iii) Attempt to expand and equate coefficients

M1

$(13 = R \cosh \ln a = R(a^2+1)/2a$
 $5 = R \sinh \ln a = R(a^2-1)/2a$)

Attempt to eliminate R (or a) to set up a solvable equation in a (or R)

M1

SC
If exponential definitions used,
 $8e^x + 18e^{-x} = Re^x/a + Rae^{-x}$ and same scheme follows

Get $a = 3/2$ (or $R = 12$)

A1

Replace for a (or R) in relevant equation to set up solvable equation in R (or a)

M1

Get $R=12$ (or $a = 3/2$)

A1

Ignore if $a=2/3$ also given

(iv) Quote/derive $(\ln^3/2, 12)$

B1√

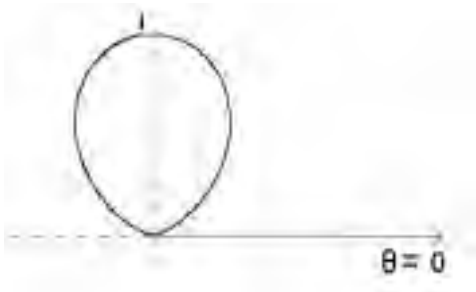
On their R and a

B1√

9(i) Use $\sin \theta \cdot \sin^{n-1} \theta$ and parts

M1

Reasonable attempt with 2 parts, one yet to be integrated

Get	A1	Signs need to be carefully considered
$-\cos\theta.\sin^{n-1}\theta+(n-1)\int\sin^{n-2}\theta.\cos^2\theta d\theta$		
Replace $\cos^2 = 1 - \sin^2$	M1	
Clearly use limits and get AG	A1	
(ii) (a) Solve for $r=0$ for at least one θ	M1	θ need not be correct
Get $(\theta) = 0$ and π	A1	Ignore extra answers out of range
	B1	General shape (symmetry stated or approximately seen)
		
	B1	Tangents at $\theta=0, \pi$ and max r seen
(b) Correct formula used; correct r	M1	May be $\int r^2 d\theta$ with correct limits
Use $6I_6 = 5I_4, 4I_4 = 3I_2$	M1	At least one
Attempt I_0 (or I_2)	M1	($I_0 = \frac{1}{2}\pi$)
Replace their values to get I_6	M1	
Get $5\pi/32$	A1	
Use symmetry to get $5\pi/32$	A1	May be implied but correct use of limits must be given somewhere in answer
Or		
Correct formula used; correct r	M1	
Reasonable attempt at formula		
$(2i\sin\theta)^6 = (z - 1/z)^6$	M1	
Attempt to multiply out both sides		
(7 terms)	M1	
Get correct expansion	A1	
Convert to trig. equivalent and integrate their expression	M1	cwo
Get $5\pi/32$	A1	
Or		
Correct formula used; correct r	M1	
Use double-angle formula and attempt to cube (4 terms)	M1	
Get correct expression	A1	
Reasonable attempt to put $\cos^2 2\theta$ into integrable form and integrate	M1	
Reasonable attempt to integrate $\cos^3 2\theta$ as e.g. $\cos^2 2\theta.\cos 2\theta$	M1	cwo
Get $5\pi/32$	A1	