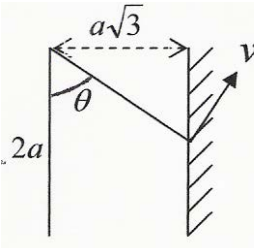
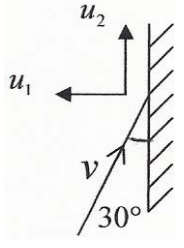
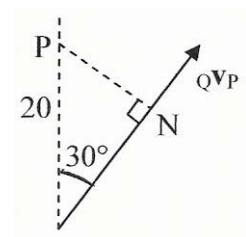
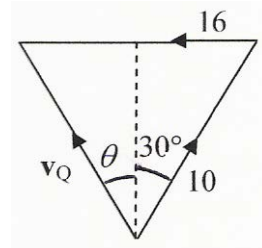
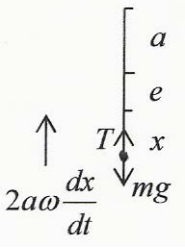
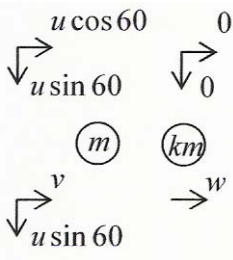
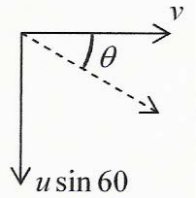


Question Number	Scheme	Marks
1.	<p>(a) <math>\frac{1}{2} \frac{dv}{dt} = \frac{1}{2} g - 2v</math>  <math>\Rightarrow 5 \frac{dv}{dt} = 49 - 20v</math> (*)</p> <p>(b) <math>\int \frac{5dv}{49 - 20v} = \int dt</math> (separate variables)  <math>\frac{-5}{20} \ln(49 - 20v) = t + c</math>  <math>t = 0, v = 0 \Rightarrow c = -\frac{1}{4} \ln 49</math> (attempt to get c)  <math>t = \frac{1}{4} \ln \left( \frac{49}{49 - 20v} \right)</math>  <math>t = 1 : 1 = \frac{1}{4} \ln \left( \frac{49}{49 - 20v} \right)</math> (correct use of logs/exp)  <math>\rightarrow v \approx 2.41 \text{ms}^{-1}</math> or <math>2.4 \text{ms}^{-1}</math></p>	<p>M1 A1 (2)</p> <p>M1 A1 M1 M1 A1 (5) <b>Total 7 marks</b></p>
2.	<p>(a) Energy: <math>\frac{1}{2} m \left( \frac{37ga}{5} - v^2 \right) = mg \cdot 2a(1 - \cos \theta)</math>  Using <math>\theta = \frac{\pi}{3}</math> &amp; solve: <math>\rightarrow v = \sqrt{\frac{27ga}{5}}</math> (*)</p> <p>(b) Impact: <math>u_1 = ev \sin 30</math>  KE loss = <math>\frac{1}{2} m (v^2 \sin^2 30 - e^2 v^2 \sin^2 30)</math>  <math>\left[ +\frac{1}{2} m v^2 \cos^2 30 - \frac{1}{2} m u_2^2 \right] = \frac{3mga}{5}</math>  [Using <math>u_2 = v \cos 30</math> if necessary &amp; ]  reducing to equation in (m, g, a) e alone  <math>\frac{3mga}{5} = \frac{1}{2} m \cdot \frac{27ga}{5} \cdot \frac{1}{4} (1 - e^2)</math>  Solve for e: <math>\rightarrow e = \frac{1}{3}</math></p>	 <p>M1 A1 M1 A1 (4)</p>  <p>M1 A1 M1 A1 A1 M1 A1 (7) <b>Total 11 marks</b></p>

Question Number	Scheme	Marks
3.	<p>(a)</p> <p>(i) <math>\mathbf{v}_Q = \mathbf{v}_Q + \mathbf{v}_P</math>  <math> \mathbf{v}_Q ^2 = (10 \cos 30) ^2 + (16 - 10 \sin 30) ^2</math>  <math>= 75 + 121</math>  <math>\Rightarrow  \mathbf{v}_Q  = 14 \text{ms}^{-1}</math></p> <p>(ii) <math>\tan \theta = \frac{16 - \sin 30}{10 \cos 30}</math> (o.e.)  <math>\theta \approx 51.8^\circ, \Rightarrow \text{bearing } \underline{308^\circ}</math> (nearest degree)</p> <p>(b) At nearest approach: <math>PN = 20 \sin 30</math>  <math>= \underline{10 \text{ km}}</math></p> <p>(c) <math>\text{Time} = \frac{20 \cos 30}{10} \approx 1.732 \text{ hrs}</math>  <math>\Rightarrow \underline{\text{Time} \approx 4.44 \text{ pm}}</math> (AWRT)</p> <p><u>Alternatives</u></p> <p>(a) Use of cosine rule in velocity vector triangle.</p> <p>(b) &amp; (c) Use of scalar product of relative velocity and relative position or differentiating magnitude of relative position vector squared to find the minimum distance and time at which it occurs.</p>	<p>M1 A1 A1 M1 A1, A1 (6)</p> <p>M1 A1 A1 (3)</p> <p>M1 A1 A1 (3)</p> <p><b>Total 12 marks</b></p>



Question Number	Scheme	Marks
4.	<p>(a) R(↓) <math>m \frac{d^2x}{dt^2} = mg - T - 2m\omega \frac{dx}{dt}</math> (<u>4</u> terms)</p> $m \frac{d^2x}{dt^2} = mg - \frac{2m\omega^2 a}{a} (e + x) - 2m\omega \frac{dx}{dt}$ $\rightarrow \frac{d^2x}{dt^2} + 2\omega \frac{dx}{dt} + 2\omega^2 x = 0 \quad (*)$  <p>(b) <math>x = e^{-\omega t} (A \cos \omega t + b \sin \omega t)</math>  <math>t = 0, x = 0 \Rightarrow \underline{A = 0}</math>  <math>\frac{dx}{dt} = -\omega e^{-\omega t} \cdot B \sin \omega t + e^{-\omega t} \cdot B \omega \cos \omega t</math> (use of product rule)  <math>t = 0, \frac{dx}{dt} = U : U = B \omega \Rightarrow \underline{B = \frac{U}{\omega}}</math></p> <p>(c) <math>\frac{dx}{dt} = -U e^{-\omega t} \sin \omega t + U e^{-\omega t} \cos \omega t = 0</math>  <math>\Rightarrow \tan \omega t = 1</math> (solve for <math>\tan \omega t</math>)  <math>\Rightarrow t = \underline{\frac{\pi}{4\omega}}</math></p>	<p>M1 A1          ↓          M1          ↓          M1 A1          (5)</p> <p>B1          M1          M1 A1          (4)</p> <p>M1          M1          A1          (3)</p> <p><b>Total 12 marks</b></p>

Question Number	Scheme	Marks
5.	<p>(a) <math>CLM(\leftrightarrow): mu \cos 60 = mv + kmw</math>  <math>NLI: \frac{1}{2}u \cos 60 = w - v</math>                      Solve for w: <math>(1+k)w = \frac{1}{2}u\left(1 + \frac{1}{2}\right)</math>  <math>\Rightarrow w = \frac{3u}{4(k+1)} \quad (*)</math></p> <p>(b) Solve for v <math>\rightarrow v = \frac{u(2-k)}{4(k+1)}</math>  <math>\tan \theta = 2\sqrt{3} = \frac{u \sin 60}{v}</math>  <math>= \frac{u\sqrt{3}}{2} \cdot \frac{4(k+1)}{u(2-k)}</math>                      Solve k: <math>\rightarrow k = \frac{1}{2}</math></p> <p>(c) <math>k = \frac{1}{2} \Rightarrow v = \frac{u}{4}, w = \frac{u}{2}</math>  <math>KE \text{ loss} = \frac{1}{2}mu^2 - \left( \frac{1}{2}m \cdot \frac{u^2}{16} + \frac{1}{2}m \cdot \frac{3u^2}{4} + \frac{1}{2} \cdot \frac{1}{2}m \cdot \frac{u^2}{4} \right)</math>  <math>= \frac{1}{2}mu^2 \left( 1 - \frac{1}{16} - \frac{3}{4} - \frac{1}{8} \right)</math>  <math>= \frac{1}{32}mu^2</math></p>	 <p>M1 A1                      M1 A1                      M1                      A1                      (6)</p>  <p>M1 A1                      M1 A1                      M1 A1                      (6)</p> <p>B1                      M1 A1                      A1                      (4)</p> <p><b>Total 16 marks</b></p>

Question Number	Scheme	Marks
6.	<p>(a) PE of R = <math>-\sqrt{2}mga \cos 2\theta</math> (+c) (1)</p> <p>PE of LH mass = <math>-\frac{3}{2}mg(2a - 2a \sin(45 + \theta))</math> (+c) (2)</p> <p>PE of RH mass = <math>-\frac{3}{2}mg(2a - 2a \sin(45 - \theta))</math> (+c) (3)</p> <p><math>V = (1) + (2) + (3)</math> (in terms of <math>\theta</math> etc.)</p> <p><math>= -\sqrt{2}mga \cos 2\theta - \frac{3}{2}mg[4a - a\sqrt{2}(\cos \theta + \sin \theta + \cos \theta - \sin \theta)]</math></p> <p><math>= -\sqrt{2}mga \cos 2\theta - \frac{3}{2}mga(-2\sqrt{2} \cos \theta + 4)</math></p> <p><u><math>= \sqrt{2}mga(3 \cos \theta - \cos 2\theta) + \text{constant}</math></u> (*)</p> <p>(b) <math>\frac{dV}{d\theta} = \sqrt{2}mga(-3 \sin \theta + 2 \sin 2\theta)</math></p> <p><math>\frac{dV}{d\theta} = 0 \Rightarrow 2 \sin 2\theta - 3 \sin \theta = 0</math></p> <p><math>\Rightarrow \sin \theta(4 \cos \theta - 3) = 0</math></p> <p><u><math>\Rightarrow \theta = 0</math>, or <math>\theta = \pm \arccos \frac{3}{4}</math> (= <math>\pm 0.723</math>)</u></p> <p>(c) <math>\frac{d^2V}{d\theta^2} = \sqrt{2}mga(-3 \cos \theta + 4 \cos 2\theta)</math></p> <p><math>\cos \theta = \frac{3}{4} : \frac{d^2V}{d\theta^2} = \sqrt{2}mga\left(-3 \cdot \frac{3}{4} + 4\left(2 \cdot \frac{9}{16} - 1\right)\right)</math></p> <p><math>= \sqrt{2}mga\left(-\frac{9}{4} + \frac{1}{2}\right)</math></p> <p><u><math>&lt; 0 \therefore \text{Unstable}</math></u></p>	<p>B1</p> <p>M1 A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (7)</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1, A1 (6)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p><b>Total 17 marks</b></p>