



General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics

MPC1

Unit Pure Core 1

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

2

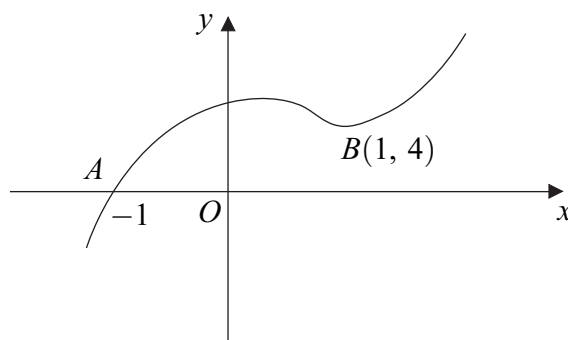
- 1 The point A has coordinates $(6, -4)$ and the point B has coordinates $(-2, 7)$.
- (a) Given that the point O has coordinates $(0, 0)$, show that the length of OA is less than the length of OB . (3 marks)
- (b) (i) Find the gradient of AB . (2 marks)
- (ii) Find an equation of the line AB in the form $px + qy = r$, where p , q and r are integers. (3 marks)
- (c) The point C has coordinates $(k, 0)$. The line AC is perpendicular to the line AB . Find the value of the constant k . (3 marks)
-

- 2 (a) Factorise $x^2 - 4x - 12$. (1 mark)
- (b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes. (4 marks)
- (c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where p and q are positive integers. (2 marks)
- (ii) Hence find the minimum value of $x^2 - 4x - 12$. (1 mark)
- (d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$. Find an equation of the new curve. You need not simplify your answer. (2 marks)
-

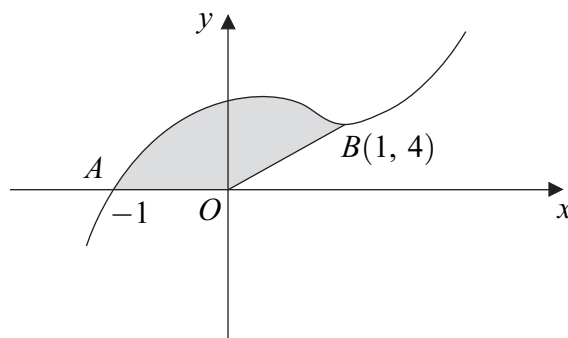
- 3 (a) (i) Simplify $(3\sqrt{2})^2$. (1 mark)
- (ii) Show that $(3\sqrt{2} - 1)^2 + (3 + \sqrt{2})^2$ is an integer and find its value. (4 marks)
- (b) Express $\frac{4\sqrt{5} - 7\sqrt{2}}{2\sqrt{5} + \sqrt{2}}$ in the form $m - \sqrt{n}$, where m and n are integers. (4 marks)



- 4 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



- (a) Given that $y = x^5 - 3x^2 + x + 5$, find:
- (i) $\frac{dy}{dx}$; (3 marks)
 - (ii) $\frac{d^2y}{dx^2}$. (1 mark)
- (b) Find an equation of the tangent to the curve at the point $A(-1, 0)$. (2 marks)
- (c) Verify that the point B , where $x = 1$, is a minimum point of the curve. (3 marks)
- (d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



- (i) Find $\int_{-1}^1 (x^5 - 3x^2 + x + 5) dx$. (5 marks)
- (ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB . (2 marks)

Turn over ►



- 5** The polynomial $p(x)$ is given by $p(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.
- (a)** When $p(x)$ is divided by $x + 2$, the remainder is -150 .
Show that $2c - d + 65 = 0$. *(3 marks)*
- (b)** Given that $x - 3$ is a factor of $p(x)$, find another equation involving c and d . *(2 marks)*
- (c)** By solving these two equations, find the value of c and the value of d . *(3 marks)*
-

- 6** A rectangular garden is to have width x metres and length $(x + 4)$ metres.
- (a)** The perimeter of the garden needs to be greater than 30 metres.
Show that $2x > 11$. *(1 mark)*
- (b)** The area of the garden needs to be less than 96 square metres.
Show that $x^2 + 4x - 96 < 0$. *(1 mark)*
- (c)** Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*
- (d)** Hence determine the possible values of the width of the garden. *(1 mark)*



7 A circle with centre C has equation $x^2 + y^2 + 14x - 10y + 49 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

(b) Write down:

(i) the coordinates of C ;

(ii) the radius of the circle. (2 marks)

(c) Sketch the circle. (2 marks)

(d) A line has equation $y = kx + 6$, where k is a constant.

(i) Show that the x -coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$. (2 marks)

(ii) The equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ has equal roots. Show that

$$12k^2 - 7k - 12 = 0 \quad (3 \text{ marks})$$

(iii) Hence find the values of k for which the line is a tangent to the circle. (2 marks)

