General Certificate of Education
June 2006
Advanced Subsidiary Examination

**MATHEMATICS** **MPC1**
**Unit Pure Core 1**

Monday 22 May 2006 9.00 am to 10.30 am

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| **For this paper you must have:**<br>• an 8-page answer book<br>• the **blue** AQA booklet of formulae and statistical tables<br>You must **not** use a calculator. | |

Time allowed: 1 hour 30 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** The point $A$ has coordinates $(1, 7)$ and the point $B$ has coordinates $(5, 1)$.

(a) (i) Find the gradient of the line $AB$. *(2 marks)*

(ii) Hence, or otherwise, show that the line $AB$ has equation $3x + 2y = 17$. *(2 marks)*

(b) The line $AB$ intersects the line with equation $x - 4y = 8$ at the point $C$. Find the coordinates of $C$. *(3 marks)*

(c) Find an equation of the line through $A$ which is perpendicular to $AB$. *(3 marks)*

**2** (a) Express $x^2 + 8x + 19$ in the form $(x + p)^2 + q$, where $p$ and $q$ are integers. *(2 marks)*

(b) Hence, or otherwise, show that the equation $x^2 + 8x + 19 = 0$ has no real solutions. *(2 marks)*

(c) Sketch the graph of $y = x^2 + 8x + 19$, stating the coordinates of the minimum point and the point where the graph crosses the $y$-axis. *(3 marks)*

(d) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 19$. *(3 marks)*

**3** A curve has equation $y = 7 - 2x^5$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

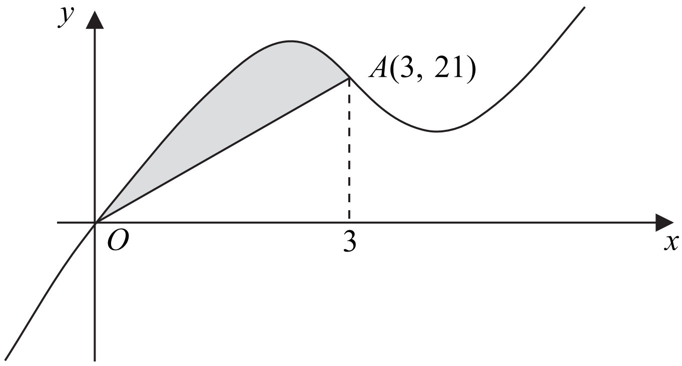
(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether $y$ is increasing or decreasing when $x = -2$. *(2 marks)*

**4** (a) Express $\left(4\sqrt{5} - 1\right)\left(\sqrt{5} + 3\right)$ in the form $p + q\sqrt{5}$, where $p$ and $q$ are integers. *(3 marks)*

(b) Show that $\dfrac{\sqrt{75} - \sqrt{27}}{\sqrt{3}}$ is an integer and find its value. *(3 marks)*

**5** The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.




The curve crosses the $x$-axis at the origin $O$ and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a stationary point when $x = 2$ and find the $x$-coordinate of the other stationary point. *(4 marks)*

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\, dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\, dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line $OA$. *(3 marks)*

**6** The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, $r$, when $p(x)$ is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express $p(x)$ in the form

$$(x - 2)(x^2 + ax + b) + r$$

where $a$, $b$ and $r$ are constants. *(4 marks)*

**Turn over for the next question**

**7** A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where $p$ is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where $k$ is a constant.

(i) Show that the $x$-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

*(3 marks)*

(ii) Find the values of $k$ for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when $k$ takes either of the values found in part (c)(ii). *(1 mark)*

**END OF QUESTIONS**